

Computer Science 4602
Fall 2022
Practice Quiz 2

You have 50 minutes. Answer all of the questions on the exam. Circle the letter of the best answer to the multiple-choice problem (marked [MC]), even if no answer is ideal. You may use one prepared 8.5×11 sheet of paper during the exam. *Check your work.*

1. Write a clearly legible T to the **left** of each of the following that is true, and a clearly legible F to the **left** of each that is false.
 - (a) Every infinite language is uncomputable.
 - (b) Every finite language is computable.
 - (c) Every partially computable language is computable.
 - (d) Every computable language is partially computable.
 - (e) Turing machines are capable of computing all languages.
 - (f) A Turing machine with 2 tapes can compute a language that cannot be computed by any Turing machine with 1 tape.
 - (g) The Halting Problem is conjectured to be uncomputable, but that conjecture has not been proved.

2. [MC] One way to prove that a set A is uncomputable is to show that
 - (a) \overline{A} is partially computable.
 - (b) $\overline{A} \leq_m \overline{\text{HLT}}$.
 - (c) $\text{HLT} \leq_m A$.
 - (d) $A \leq_m \text{HLT}$.

3. If p is a program, define $L(p)$ to be the set $\{x \mid p(x) \cong 1\}$. Let $A = \{p \mid L(p) \text{ is a finite set}\}$.
- (a) Is A a finite set?
 - (b) Is A computable? Justify your answer. You will receive no points for a yes or no answer without convincing justification.
4. Let $B = \{n \mid n \text{ is a positive integer that can be expressed as the sum of two prime numbers}\}$. For example, $8 \in B$ since $8 = 5 + 3$. Is B computable? Justify your answer. You will receive no points for a yes or no answer without convincing justification.

5. Suppose A is the set of programs $\{p \mid \text{the last character of program } p \text{ is a right brace}\}$ and $B = \{p \mid p(0) \uparrow\}$. Give a mapping reduction from A to B .

6. Suppose $A = \{p \mid p(0) = 5\}$ and $B = \{p \mid p(0) = 10\}$. Give a mapping reduction from A to B . Be sure that you know what properties the reduction needs to have before you start to describe the reduction.

7. Is relation \leq_m transitive? That is, if $x \leq_m y$ and $y \leq_m z$, is it necessarily true that $x \leq_m z$?

8. Is relation \leq_m symmetric? That is, if $x \leq_m y$ is it necessarily true that $y \leq_m x$?

9. Is $\{p \mid p(0) \downarrow\}$ partially computable?

10. What is the definition of a mapping reduction from A to B ?

11. Does $\{p \mid p(0) \cong p(1)\}$ respect equivalence? Justify your answer.

12. Is $\{p \mid p(0) \cong p(1)\}$ computable? Justify your answer.

13. Give a mapping reduction from $\{p \mid p(0) \cong 3\}$ to $\{p \mid p(1) \cong 3\}$.

14. Give a mapping reduction from $\{n \mid n \text{ is a prime integer}\}$ to $\{n \mid n \text{ is an integer that is a perfect square}\}$.