1. The Partition Problem (PP) is described in Section 13 of the notes. Give a polynomial-time reduction from PP to the Subset Sum Problem.

The Partition Problem is a special case of the Subset Sum Problem where the desired sum is exactly half of the total sum of the numbers in the input. If the total sum is odd, then the answer for the Partition Problem is no, and that is easy to deal with.

The reduction *f* starts by computing the sum *K* of $x\_{1}…, x\_{n}$.

 If *K* is even, then *f* ($x\_{1}…, x\_{n}$) = ($x\_{1}…, x\_{n}$, *K*/2)

 If *K* is odd, then *f* ($x\_{1}…, x\_{n}$) = ((1), 2)

In the second case, *f* ($x\_{1}…, x\_{n}$) yields an input to SSP for which the answer is no. You cannot select values from list (1) whose sum is 2.

1. Exercise set 0911 defines isomorphic graphs. Suppose that $G$ and $H$ are simple graphs. Say that $H$ is isomorphic to a subgraph of $G$ provided it is possible to remove zero or more vertices and zero or more edges from $G$ and get a graph that is isomorphic to $H$. (When you remove a vertex $v$, you must also remove all edges that are incident on $v$.) The Subgraph Isomorphism Problem (SIP) is the following decision problem.

**Input.** Simple graphs $G$ and $H$.

**Question.** Is $H$ isomorphic to a subgraph of $G$?

Prove that SIP is NP-complete. (**Hint.** Reduce from the Clique Problem.)

A. We need to show that SIP is in NP. Here is a polynomial-time evidence checker for SIP.

**Evidence:** If *H* has *N* vertices then the evidence is a list $u\_{1},…,u\_{N}$ of selected vertices of *G* and an ordering $v\_{1},…, v\_{N}$ of the vertices of H.

**Requirement:** For *i*, *j* = 1,…, *N* where $i \ne j$, if $\{v\_{i},v\_{j}\}$ is an edge in *H* then $\left\{u\_{i},u\_{j}\right\}$ is an edge in *G*. You don’t need $\leftrightarrow $ because you are looking for a subgraph of *G*, and not all edges in *G* need to be included.

B. We reduce the Clique Problem to SIP.

*G* has a clique of size *k* if and only if a complete graph of *k* vertices is isomorphic to a subgraph of *G*.

A polynomial-time mapping reduction from CP to SIP is

 $f\left(G,k\right)=(G, K\_{k})$ where $K\_{k}$ is a complete graph of *k* vertices.