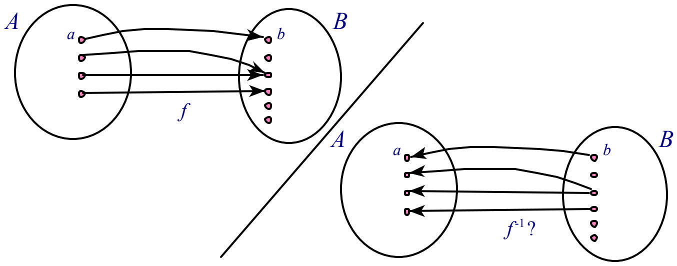
## Inverse of a Function

- Under what circumstances does a function have an inverse?
- That is, when does a function $f: A \to B$ have an inverse $f^{-1}: B \to A$ such that $f^{-1}(f(x)) = x$?




## Inverses and Bijections

- A function $f: A \to B$ has an inverse $f^{-1}: B \to A$ if and only if $f$ is a bijection
- On the other hand, suppose you were willing to have an inverse defined not on all of $B$, but on a subset of $B$
  - Which subset?
  - The *Range* of $f$
- Under what circumstances can you find an inverse by restricting to the range?
  - If the function is 1-1

## Sequences

- A sequence is an ordered collection of objects
- It's a tuple without the parentheses
- but it can be finite or infinite
- Sequences do not always start with element 1; they can start with their 0th element, or 2nd element, etc...
- Here is a five element sequence:
  - $a_1, a_2, a_3, a_4, a_5$
- Here is an infinite sequence:
  - $a_2, a_3, a_4, a_5, \ldots$

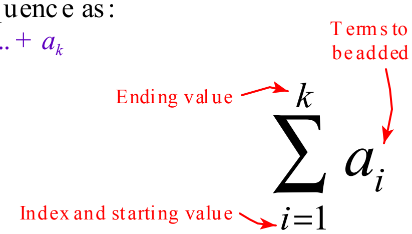
## Formula for a Sequence

- Sometimes sequences are given by a formula:
  - $a_k = k^2 - 2$, for $k \ge 1$
  - sequence: $-1, 2, 7, 14, \ldots$
  - It's an infinite sequence
- And sometimes we are given a sequence and are asked to find a formula for that sequence:
  - 1, 3, 6, 10, 15, 21, 28, 38, 45, 55, ...
  - But that's a topic for another day

## Sum of a Sequence

- Sometimes we're going to want to add together the terms of a sequence
- If $a_1$, $a_2$, $a_3$, $a_4$ ..., $a_k$ is a sequence, then we write the sum of this sequence as:
  - $a_1 + a_2 + a_3 + a_4 + ... + a_k$

$$= \sum_{i=1}^{k} a_i$$




## Your Turn

Evaluate: $\sum_{k=1}^{5} (k^2 + 1) = 60$

Evaluate: $\sum_{k=1}^{6} (k^2 + 1) = 97$

Evaluate: $\sum_{j=1}^{3} \sum_{k=1}^{4} (k^2 + j) = 114$

Evaluate: $\sum_{j=1}^{5} \sum_{k=1}^{j} (k^2 + 1) = 120$

## Some Sum Formulas

- Please commit the following to memory (p. 76)
  - Sum of a geometric series: $\sum_{k=0}^{n} ar^k = \frac{ar^{k+1} - a}{r - 1}$
  - Sum of the counting numbers: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
  - Sum of the squares: $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

## Double Sums

- Compute the following sums:

$$\sum_{i=1}^{5} \sum_{j=1}^{5} ij = 225$$

$$\sum_{i=1}^{5} \sum_{j=1}^{i} ij = 140$$

$$\sum_{i=1}^{5} \sum_{j=1}^{5-i} 2^i + j^2 = 102$$

## Your Turn Again

Evaluate: $\sum_{i=1}^{30} (i^2 + 3i)$

Evaluate: $\sum_{c=21}^{40} c^2$

Evaluate: $\sum_{4 \le u \le 9} 3^u$