Ordered pairs and *n*-tuples

- The *ordered pair* (a, b) is a collection that has a as its first element and b as its second element
  - Note how this is different from a set; in sets the elements are not ordered
  - The set $\{1, 2\}$ is the same as the set $\{2, 1\}$, but the ordered pairs $(1, 2) \neq (2, 1)$
- The *ordered triple* (a, b, c) is a collection with a as its first element, b as its second element and c as its third element.
- The *ordered n-tuple* $(a_1, \ldots, a_n)$ is the collection which has a_i as its ith element, for $1 \leq i \leq n$

Cartesian Products

- The Cartesian product gives us a way to "multiply" sets together
- If $A, B, \ldots, Q$ are sets, then the Cartesian product $A \times B \times \cdots \times Q$ is the set $\{(a, b, \ldots, q) \mid a \in A, \ldots, q \in Q\}$
- Note that the Cartesian product of k sets is a set of ordered k-tuples
- What is $\{x, y\} \times \{p, q\}$
  - $\{(x, p), (x, q), (y, p), (y, q)\}$
- What is $\{1, 2, 3\} \times \{a, b\} \times \{5, 6\}$?
  - $\{(1, a, 5), (1, a, 6), (1, b, 5), (1, b, 6), (2, a, 5), (2, a, 6), (2, b, 5), (2, b, 6), (3, a, 5), (3, a, 6), (3, b, 5), (3, b, 6)\}$

A Familiar Cartesian Product

- You've probably heard the term "Cartesian coordinates"
- It usually refers to the (x, y)-coordinate system we use when graphing in the plane
- or (x, y, z)-coordinates when graphing in space
- If we let $\mathbb{R}$ denote all real numbers (the number line), then the plane is $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$
- and space is $\mathbb{R} \times \mathbb{R} \times \mathbb{R} = \mathbb{R}^3$

Union of Sets

- The *union* of A and B is the set that contains all elements found in either A or B
- $A \cup B = \{x \mid x \in A \lor x \in B\}$
- Eg: $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{1, 4, 7\}$.
  - What is $A \cup B$?
  - What is $A \cup C$?
  - What is $A \cup B \cup C$?
- What is $A \times B \cup C$
  - It depends on where you put the parentheses!
  - $(A \times B) \cup C = \{(1, 3), (2, 3), (3, 3), (1, 4), (2, 4), (3, 4), 1, 4, 7\}$
  - $A \times (B \cup C) = \{(1, 1), (1, 3), (1, 4), (1, 7), (2, 1), (2, 3), (2, 4), (2, 7), (3, 1), (3, 3), (3, 4), (3, 7)\}$

Intersection of Sets

- The *intersection* of *A* and *B* is the set that contains all elements found in both *A* and *B*
- *A*1*B* = {*x* | *x* 0 *A* v *x* 0 *B*}
- Eg: *A* = {1, 2, 3}, *B* = {3, 4}, *C* = {1, 4, 7}.
  - What is *A*1*B*?
  - What is *A*1*C*?
  - What is *A*1*B*1*C*?
- What is *A*×*B*1*C*
  - (*A*×*B*)1*C* = i
  - *A*×(*B*1*C*) = {(1, 4), (2, 4), (3, 4)}

Set Difference

- The *difference* of *A* and *B* is the set that contains all elements found in *A*, but not *B*
- *A* ! *B* = {*x* | *x* 0 *A* v *x* ó *B*}
- Eg: *A* = {1, 2, 3}, *B* = {3, 4}, *C* = {1, 4, 7}.
  - What is *A* ! *B*?
  - What is *A* ! *C*?
  - What is *A* ! *B* ! *C*?
    - Here it depends on where you put the parentheses!
    - (*A* ! *B*) ! *C* = {2}
    - *A* !(*B* ! *C*) = {1, 2}
  - What is *A*×*B* ! *C*
    - (*A*×*B*)1*C* = i
    - *A*×(*B*1*C*) = {(1, 4), (2, 4), (3, 4)}

Complement of a Set

- The complement of a set is the set of all elements that are not in that set.
  - Does this make sense?
  - What's missing?
- Before defining the complement of a set, we must have the notion of a *universal set*
  - The universal set is the set of all elements under consideration. This must be made clear.
- This is analogous to the *universe of discourse* for predicates
- Alltogether: *I will not write "compliment" of a set*

Complement of a Set

- Suppose *U* is the universal set. Then for a set *A*,

$$\overline{A} = \{x \in U | x \notin A\}$$

- Thus *U* – *A* is another way of writing the complement
- Suppose *U* is the set of all integers
  - *A* = {*x* | *x* is an even integer}
    - complement = set of odd integers
  - *B* = {*x* | *x* is a multiple of 3}
    - complement = set of integers which leave a remainder of 1 or 2 when divided by 3

Set Identities

P Several set identities are give on page 49:

Ac i = A	A1U = A
AcU = U	A1 i = i
AcA = A	A1A = A
...	
A1(BcC) = (A1B)c(A1C)	Ac(B1C) = (AcB)1(AcC)

P How would one prove such identities?

P Three ways:

< Membership Tables

< Use of simpler identities

< In general, to show two sets are equal, it is enough to show that each set is a subset of the other set

Membership Tables

P For each simple set in the expression, make a column that will indicate whether an element is (1) or is not (0) in that set

P For more complicated columns, combine the simpler columns

P For example, let us prove one of DeMorgan's laws: *The complement of the union is the intersection of the complements*

Membership Tables

A	B	AcB	$\overline{A\text{c}B}$	$\overline{A}$	$\overline{B}$	$\overline{A}$1$\overline{B}$
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1

Prove (A!B)1C f C ! B

P This time, what would we require of the membership table?

< That every "1" on the left-hand side has a corresponding "1" on the right-hand side.

< But there could be more "1s" on the RHS than the LHS

A	B	C	A!B	(A!B)1C	C!B
1	1	1	0	0	0
1	1	0	0	0	0
1	0	1	1	1	1
1	0	0	1	0	0
0	1	1	0	0	0
0	1	0	0	0	0
0	0	1	0	0	1
0	0	0	0	0	0

Every "1" in this column

Has a corresponding "1" in that column, proving the desired subset relationship.