## Negating Expressions

- We use DeMorgan's laws to negate conjunctions and disjunctions:
  - ¬(*p* ∨ *q*) ] ¬*p* w ¬*q*
  - ¬(*p* w *q*) ] ¬*p* ∨ ¬*q*
- And to negate quantifiers, we use the following:
  - ¬œ*x* P(*x*) ] ›*x* ¬P(*x*)
  - ¬›*x* P(*x*) ] œ*x* ¬P(*x*)
- For example, we simplify: ¬(›*x* P(*x*) w œ*x* Q(*x*))
  - ¬(›*x* P(*x*) w œ*x* Q(*x*)) ] ¬›*x* P(*x*) ∨ ¬œ*x* Q(*x*)
  - ] ¬›*x* P(*x*) ∨ ¬œ*x* Q(*x*) ] œ*x* ¬P(*x*) ∨ ›*x* ¬Q(*x*)
- Compare first and last expressions if P(*x*) means "*x* is a dictator" and Q(*x*) means "*x* is free"

## Sets

- Sets are collections of objects.
- The objects in a set are called its *elements*. If *a* is an element of set *A*, we write *a* 0 *A*
- A set can be described in two ways:
  - Explicitly, by listing its elements within curly braces:
    - {a, b, c, d, e}
    - {Raleigh, Albany, Austin, ..., Sacramento}
    - {1, 2, 3, Albert Einstein, North Pole, 4, 5}
  - With *set builder* notation
    - {*x*| *x* > 10}
    - {*p*| *p* is prime and *p* is one more than a multiple of 4}
    - {*a*| *a* is a subset of {1, 2, 3} }
  - Note the need for a universe of discourse when using set builder notation

## The Empty Set

- There is a unique set which contains no elements
- It's called the *empty set*
- It is denoted by "i "
- ...or "{}"

- It is a deceptively important set. Don't forget about it!
- It's the set analogue of the number "0"

## Cardinality of a Set

- The *cardinality* of a set is simply the number of elements of that set
- For a finite set with *n* elements, the cardinality of that set is *n*
- If a set does not have a finite number of elements, then we say it is of infinite cardinality
  - We will see later that some infinities are bigger than others
- We denote the cardinality of a set *S* by |*S*|
- What is |i |?
  - 0

## Equality of Sets

PTwo sets are equal if they have exactly the same elements

PNote that this implies that if two sets are equal, then the have the same cardinality

PThus, the following pairs are not equal:

< {1, {2} } ... { {1}, 2}
< 2 ... {2}
< {*x* | $x^2$ = 4} ... {2}

## Subsets

PIf *A* and *B* are two sets, then we call *A* a *subset* of *B* if every element of *A* is also an element of *B*

< *A* f *B* ø ( *x*0 *A* 6 *x*0 *B* )

PWe write *A* f *B*

PIf *A* f *B*, but *A* ... *B*, then we can write *A* d *B* to denote that *A* is a *proper subset* of *B*

Let *S* = {1, 2, 3} *T* = {1, 2, 3, 4}
*U* = {0, 1, 2, 3} *V* = {*y* | *y* is an integer < 4}

Which of the sets above are subsets of which other sets? Can you suggest a nice way to picture these subset relationships?

## The Power Set

The *power set* of a set *A is* the set of all subsets of *A*

PThe power set is a set all of whose elements are also sets

PFind the power set of each of the following:

< {1}
< {a, b}
< {", $, (}
< i