Predicates

- Expressions that have variables in them are not propositions
  - $x = 21$
  - " > $
  - The graph G is Eulerian, but not Hamiltonian
- Such expressions are called *predicates*, or *propositional functions*
- We could let $P(x)$ be the expression: $x = 21$
  - Then $P(5)$ is false
  - But $P(21)$ is true

Universe of Discourse

- The *universe of discourse* is the set of all values that the variables in a predicate can take on
- It must be made clear what the universe of discourse is, for every variable involved, before one can do any work with a predicate
- Let $P(x, y)$ be the assertion that "x has eaten y"
  - x could be all ECU students and y all breakfast cereals
  - x could be all bears at the zoo and y could be all species of saltwater fish
  - As you see, the universe of discourse is important

Quantifiers

- There are two kinds of quantifiers:
  - The *existential quantifier* ›
  - The *universal quantifier* œ
- Let $P(x)$ be some predicate:
  - The existential quantification ›$xP(x)$ asserts that $P(x)$ is true *for some* value x in the universe of discourse
  - The universal quantification œ$xP(x)$ asserts that $P(x)$ is true *for all* values x in the universe of discourse
- Let $P(x)$: x is the sum of the squares of two integers, where the univ. of disc. is all integers
  - Which of the following are true: ›$xP(x)$, œ$xP(x)$

Examples of Quantification

- Here are some predicates:
  - P(x): x is male
  - Q(x): x is female
  - R(x, y): x and y are the same sex
- Which of the following are true, assuming the universe of discourse is "all ECU students":
  - ›x P(x) (True)
  - ›a [P(a) ∨ Q(a)] (False)
  - ›s ›t [P(s) ∨ Q(t)] (True)
  - œz P(z) (False)
  - œx ›y R(x, y) (True)
  - ›a ›b [P(a) ∨ Q(b) ∨ R(a, b)] (False)
  - œx œy [(P(x) ∨ R(x, y)) 6 P(y)] (True)

Quantifying on More than one Variable

- Let $P(x, y)$ be the assertion that $xy = 1$, where the universe of discourse for each variable is the set of rational numbers
- Which of the following expressions are true:
  - œ*x* ›*y* *P*(*x*, *y*)
  - ›*x* œ*y* *P*(*x*, *y*)
  - ›*x* ›*y* *P*(*x*, *y*)
  - œ*x* œ*y* *P*(*x*, *y*)
  - œ*x*, (*x* ...0 6 ›*y* *P*(*x*, *y*))
- Suppose $P(x, y)$ is the assertion that *x* $ *y*, where the universe of discourse is {1, 2, 3, 4, 5, 6, 7}.

Predicates become Propositions

- Notice that predicates become propositions when its variables are quantified
- But we must quantify *all* variables in the predicate for it to become a proposition