Compound Statements

- A statement that is composed of simpler statements joined by some logical connective(s) such as "v," "w," "r," etc... is called a *compound proposition*
- A statement which is not a compound statement is called a *simple proposition*
- We are thus interested in determining the truth value (True or False) of a compound proposition given the truth values of the simple propositions of which it is composed.

Truth Tables

p	¬*p*
T	F
F	T

p	*q*	*p* v *q*
T	T	T
T	F	F
F	T	F
F	F	F

p	*q*	*p* w *q*
T	T	T
T	F	T
F	T	T
F	F	F

p	*q*	*p* 6 *q*
T	T	T
T	F	F
F	T	T
F	F	T

p	*q*	*p* r *q*
T	T	F
T	F	T
F	T	T
F	F	F

This is the *exclusive OR*

This is the *biconditional*

p	*q*	*p* ø *q*
T	T	T
T	F	F
F	T	F
F	F	T

Some Notes on the Implication

- Let *p* 6 *q* be an implication
- Then *q* 6 *p* is called the *converse* of *p* 6 *q*
- And ¬*q* 6 ¬*p* is called the *contrapositive* of *p* 6 *q*
- As we shall see, the contrapositive of an implication is equivalent to the implication itself. This is a very important and useful observation when we wish to do proofs.
- Does anyone wonder why F 6 T is True, for example?

Propositional Equivalences

- Two propositions are called *logically equivalent* if they always have the same truth value
- That is, *p* and *q* are logically equivalent if *p* ø *q* is always true
- Propositions which are always true are called *tautologies*
- Propositions which are always false are called *contradictions*
- Propositions which are sometimes true, sometimes false are called *contingencies*

Examples of Tautologies, Contradictions and Contingencies

Tautology	Contradiction	Contingency
p w *q* w ¬*p*	(*p* ø *q*) v (*p* r *q*)	*p* 6 (*q* 6 *r*)

- PWhy is this tautology always true?
- PWhy is this contradiction always false?
- PWhen is this contingency False? When is it True?

Logical Equivalence

PHow would one show that two propositions are logically equivalent?

- < Show that they have the same truth tables
- < Show that their biconditional is a tautology
- < Use some of the equivalences from page 17

p v T] *p*
p w T] T
¬(*p* w *q*)] ¬*p* v ¬*q*
etc...

p w F] *p*
p v F] F
¬(*p* v *q*)] ¬*p* w ¬*q*
etc...

Logical Equivalence

Let us show that *p* 6 *q*] ¬*p* w *q* with a truth table:

p	*q*	*p* 6 *q*	¬*p*	¬*p* w *q*
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Can you see how the "biconditional" method would work? We would want (*p* 6 *q*) ø (¬*p* w *q*) to be a tautology

Identifying Truth Tables

Can you identify the truth tables below:

p	*q*	*???*
T	T	T
T	F	F
F	T	F
F	F	F

p	*q*	*???*
T	T	F
T	F	T
F	T	T
F	F	F

p	*q*	*???*
T	T	T
T	F	F
F	T	T
F	F	F

PIt turns out that, given any truth table, we can construct an expression that gives exactly that table!

Constructing Truth Tables

- Identify each row that is true
- Construct an “and” expression that says it will be true
  - <*p* ∨ *q*
  - <¬*p* ∨ *q*
- Take the “or” of those expressions
  - <(*p* ∨ *q*)W(¬*p* ∨ *q*)
- Of course, there may be a simpler expression that is logically equiv.
  - <*q*

p	*q*	???
T	T	T
T	F	F
F	T	T
F	F	F

Constructing a 3-variable Truth Table

- Construct an expression whose truth table is that shown to the right.
- We create a conjunction for each row that must be true:
  - <(*p* ∨ *q* ∨ *r*)
  - <(*p* ∨ ¬*q* ∨ *r*)
  - <(*p* ∨ ¬*q* ∨ ¬*r*)
  - <(¬*p* ∨ ¬*q* ∨ *r*)
  - <(¬*p* ∨ ¬*q* ∨ ¬*r*)
- The disjunction of those five expressions is the answer:

<(*p*∨*q*∨*r*)W(*p*∨¬*q*∨*r*)W(*p*∨¬*q*∨¬*r*)W(¬*p*∨¬*q*∨*r*)W(¬*p*∨¬*q*∨¬*r*)

p	*q*	*r*	???
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	T