Propositions in Logic

- A *proposition* is a sentence which is unambiguously true or false.
- Propositions:
  - 4 = 2 + 2
  - 5 > 10
  - Montana is the capital of Mozambique
- Not Propositions:
  - x = 21
  - " > $
  - The graph *G* is Eulerian, but not Hamiltonian

Representing Propositions with Letters

- When discussing the rules of logic, we often denote propositions with letters, just like variables in mathematics
- We can pull out some propositions from what we were given and what we wished to prove:
  - p: *G* is planar
  - q: *G* contains a subdivision of K_5
  - r: *G* contains a subdivision of $K_{3,3}$
  - s: *G* has at least five vertices of degree at least 4

Translating to Mathematics

- Our *givens* are:
  - ¬*p*
  - ¬*s* (this is *implied* by the second given proposition)
  - *p* 6 (*q* w *r*)
  - *q* 6 *s*
- We are asked to prove:
  - *r*
- We can prove this using the mathematical rules of logical inference

But first... A little notation

Logical Symbols

Negation	¬*p*	not *p*	True when *p* is false
Conjunction	*p* ∨ *q*	*p* and *q*	True when both *p* and *q* are true
Disjunction	*p* w *q*	*p* or *q*	True when either *p* or *q* is true, or both
Implication	*p* 6 *q*	*p* implies *q* if *p*, then *q* *p* only if q 9 *implicitly*	Always true, unless *p* is true and *q* is false.

Compound Statements

- A statement that is composed of simpler statements joined by some logical connective(s) such as "v," "w," "r," etc... is called a *compound proposition*
- A statement which is not a compound statement is called a *simple proposition*
- We are thus interested in determining the truth value (True or False) of a compound proposition given the truth values of the simple propositions of which it is composed.

Truth Tables

p	¬*p*
T	F
F	T

p	*q*	*p* v *q*
T	T	T
T	F	F
F	T	F
F	F	F

p	*q*	*p* w *q*
T	T	T
T	F	T
F	T	T
F	F	F

p	*q*	*p* 6 *q*
T	T	T
T	F	F
F	T	T
F	F	T

p	*q*	*p* r *q*
T	T	F
T	F	T
F	T	T
F	F	F

This is the *exclusive OR*

This is the *biconditional*

p	*q*	*p* ø *q*
T	T	T
T	F	F
F	T	F
F	F	T