Arranging Objects with Repeats

- How many ways are there to arrange the letters of the word "LONE"?
  - < 4! = 24, because the letters were all distinct
- How many ways are there to arrange the letters of the word "NONE"?
  - < The answer is not 4! anymore.
  - < How many ways to select the first letter?
    - – 3
  - < How many ways to select the second letter?
    - – The answer depends on our selection for the first letter!
  - < So, not only is the answer not 4!, but we can't even use a straightforward multiplication for this problem at all.

Arranging Objects with Repeats

- The trick is to *color* the letters, to make them all different...
  - < "NONE" has some repeats
  - < But "NONE" has none

There are 24 arrangements of the word "NONE", because now the letters are distinct.

But if we remove the colors from the letters, those 24 arrangements are not all distinct anymore.

"NONE" = "NONE", and "ONNE" = "ONNE"

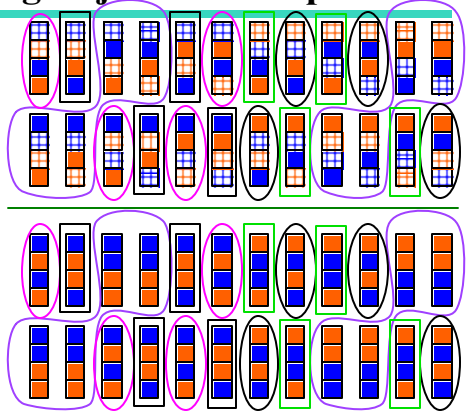
- How many distinct arrangements will there be?

Arranging Objects with Repeats

- How many ways are there to arrange the objects on the right?
- The answer is not 4!, because the objects are not all distinct
- We need a way to deal with repeats
- Our trick is to:
  - < temporarily make them all distinct
  - < then correct for overcounting
- With distinct colors, as shown here, there will be 4! = 24 ways to arrange the objects

Arranging Objects with Repeats

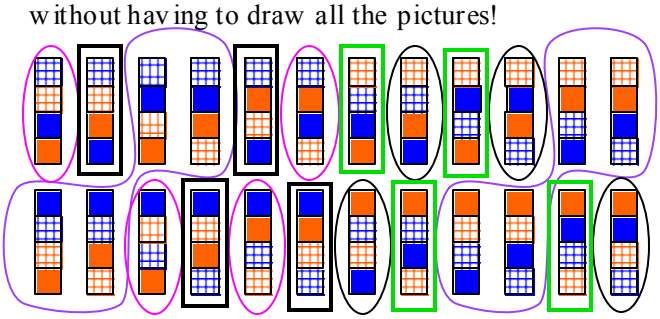
- Here are the 24 ways with distinct colors
- They are not all distinct with the original colors



Predicting the Number in each Group

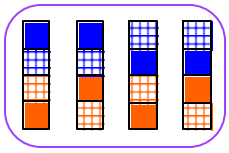
How could we have predicted that each group would have 4 arrangements in it?

We would then know that 24/4 = 6 was the answer without having to draw all the pictures!



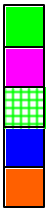
Inside the Purple Group

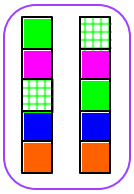
- Here is a redrawing of the purple group
- How could we have predicted that there would be 4 arrangements inside this group?
- There are two "blue" blocks.
  - < They can be arranged in 2! ways.
- There are two "red" blocks.
  - < They can be arranged in 2! ways.
- So this group will contain 2!×2! = 4 arrangements (by the product rule)
- Similarly, every group will contain 4 arrangements



How Many in each Group?

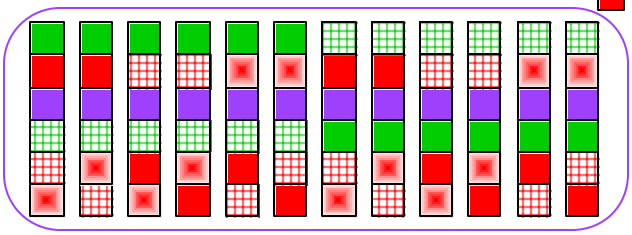
- Here is another tower of colored blocks
- Suppose we assign different colors to each block. How many arrangements?
- If we regroup those that are really the same, how many arrangements will be in each group?
- There will be 2! in each group, because there are 2 greens, which are really the same.





How Many in each Group?

- Same question for the tower of blocks to the right:
  - < When we assign distinct colors and then regroup, how many will be in each group?
  - < Did you say 3!×2! ?



Your Turn

- How many "anagrams" are there of each of the following words?
  - READ
    - 4! = 24
  - REED
    - 4! / 2! = 12
  - DEED
    - 4! / (2! × 2!) = 6
  - EPEE
    - 4! / 3! = 4
  - ANAGRAM
    - 7! / 3! = 840
  - SORTING
    - 7! = 5040

Applications of the Anagram Method

- Computing the "choose numbers."
- Given n distinct objects, the number of ways to select k of them is called "n choose k"
- For example, 6 choose 2 = 15, because there are 15 ways to select 2 objects from among 6 distinct objects
  - If we wish to select two letters from "ABCDEF", we could select AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, or EF. That's 15 ways.
- We will now derive a better way to do it.

Selecting 2 out of 6

- Let's line up the six letters "ABCDEF" and consider some ways to select two of them

A	B	C	D	E	F	Selection
Y	Y	N	N	N	N	AB
Y	N	N	N	Y	N	AE
N	N	Y	Y	N	N	CD
…						…

- Place a "Y" under each letter that is chosen, and an "N" under each that is not chosen
- Each way of selecting two letters corresponds to an anagram of the word "YYNNNN"
- Thus, the number of ways to select two letters is 6!/(2!×4!) = 15

Selecting 4 students out of 10

- A teacher wishes to select 4 students from a class of 10. In how many ways can this be done?
- Line up the students: A B C D E F G H I J
- Consider a way to select 4: Y Y Y Y N N N N N N
- Again, we see that every way to select 4 students corresponds to an anagram of "YYYYNNNNNN"
- So there are 10!/(4!×6!) = 210 ways to do this
- In general, the number of ways to select k objects from n distinct objects is $n!/(k! \times (n-k)!)$

Walking on a Grid

- Another application of anagrams:
- How many ways are there to walk from A to B on the grid below, walking only north and east?
- One way to walk: NNNNNNEEEE
- Another way: EEEENNNNNN
- Another way: NNEENNEENN
- All ways to walk have 4 "E" and 6 "N"
  < And are thus anagrams of "NNNNNNEEEE"
- The number of ways to walk is thus:
  < 10!/(4!×6!) = 210