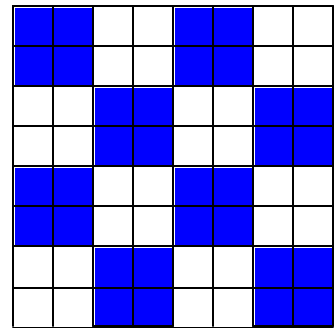


Methods of Proof

- Direct Proof
 - < Assume:
 - p
 - $p \rightarrow q$
 - < Conclude
 - q
- In proofs of this type, the bulk of the work is in showing $p \rightarrow q$
- For example:
 - < Prove that if I put 17 kings onto a 8×8 chessboard, then there must be two kings that are adjacent, either horizontally, vertically or diagonally.

Kings on a Chessboard

- Suppose I 2-color the chessboard as shown
- There are 16 2×2 regions
- If I place 17 kings into these 16 regions, then some region will get at least 2 kings
- They will be adjacent
- End of proof. •



Indirect Proof

- Indirect Proof
 - < Assume
 - $\neg q \rightarrow \neg p$
 - < Conclude
 - $p \rightarrow q$
- This is simply the assertion that an implication is logically equivalent to its contrapositive
- For example:
 - < Prove that if n is congruent to 3 (mod 4), then n is not the sum of two squares
 - p : n is congruent to 3 (mod 4)
 - q : n is not the sum of two squares

Sum of Two Squares

- Prove that if n is congruent to 3 (mod 4), then n is not the sum of two squares
 - < p : n is congruent to 3 (mod 4)
 - < q : n is not the sum of two squares
- We will show that $\neg q \rightarrow \neg p$
 - < Assume $\neg q$. Then n is the sum of two squares:
 - $n = a^2 + b^2$, where a and b are integers
 - < Let's consider the possible values of a square (mod 4):
 - Every integer is either 0, 1, 2 or 3 (mod 4)
 - The squares of those are 0, 1, 4, 9 (mod 4), which reduce to:
 - 0, 1, 0, 1 (mod 4)
 - That is, every square is congruent to either 0 or 1 (mod 4)
 - < But then the sum $a^2 + b^2$ is either 0, 1 or 2
 - < But *not* 3. So n is *not* congruent to 3 (mod 4)

Proof by Contradiction

- Proof by contradiction

- < Assume:

- $\neg p$ is False

- < Conclude

- p

- We show that if p is not true, then we obtain a contradiction. This implies that p must be true

- Example:

- < Prove that an 8×8 checkerboard with two opposite corners removed cannot be tiled with 1×2 dominos