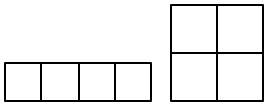
Practice Problems:

1. Complete the tree diagram. Use it to make a systematic listing of all possible arrangements of the four letters A, B, C, and D. Confirm the number of arrangements using the multiplication rule for counting.

2. Six different colored blocks are lined up in a row. How many color arrangements are possible?

3. A teacher who loves discrete math gives a workshop which causes 17 other teachers to love discrete math. Then each of those teachers teaches discrete math to 22 students, who gain such an appreciation of discrete math that they each go home and tell both of their parents an interesting discrete math fact. How many parents has the original teacher thereby influenced with her single workshop?

4. How many different colorings are possible if each small square in each of these arrangements is either shaded or left blank? Support your answers by listing systematically all of the possibilities.



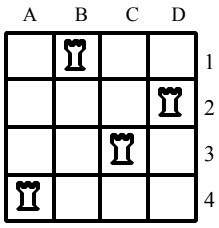
5. When Alice sends Bob a card, she always adds a string of 5 X's and/or O's after her signature, with one restriction: she never puts two O's next to each other, because Bob turns them into smiley faces, and she can't stand that. Thus, for example, "OXOXO" and "XOXXO" would be allowed, but "XOOXX" and "OOXOO" would not. Systematically list all the different ways for Alice to add X's and O's to the card. How many ways are there?

6. Start with the letters in the word *graphs*.
   a. How many ways can you select and arrange 2 of the letters? 1 letter? 1 or 2 letters? 3, 4, or 5 letters?
   b. Explain the counting techniques used in terms of the multiplication and addition rules.

7. The Perfect Spy, *Agent 006*, is sealed inside a room with no way to escape. With him in the room is a time-bomb set to explode in 9 minutes. It would take him just 30 seconds to defuse the bomb, except that it is locked inside a suitcase that has a combination lock on one of the clasps. To open the lock, he has to set three dials, each containing the digits 0-9, to the correct positions. He is able to check two of these combinations per second, thanks to his top-notch manual dexterity. Is he guaranteed to be able to open the suitcase with enough time left to defuse the bomb?

8. Make a tree diagram showing all arrangements of the letters in the word "REED". How many are there?

9. The figure to the right shows 4 rooks placed on a 4x4 "chessboard" in such a way that no rook attacks any other rook. (In chess, a rook attacks pieces which lie in the same row or column as itself.)



   a. List systematically all the ways to place 4 non-attacking rooks on a 4x4 board. Use the grids on page EX 4. (In the diagram, the rooks are at squares A-4, B-1, C-3, and D-2.)
   b. How many ways are there?
   c. Is there a way to use the multiplication rule to find this answer?

10. a. How many 3-digit numbers are there where each digit is odd?
   b. How many 3-digit numbers are there where each digit is odd and all three digits are different?
   c. How many 3-digit numbers are there where each digit is odd and the three digits appear in increasing order? (For example, 157 is such a number.) Can you answer this question using the multiplication rule?

11. In this problem, we use the word "word" to mean any sequence involving the indicated letters. For example, SUSUSU is a six letter word using only S and U, as are SSSSSS and UUUSSS.
   a. How many six letter words can you make using only S and U?
   b. How many four letter words can you make using only R, E, D?
   c. How many five letter words can you make using R, E, A, D?
   d. If you wanted to make that many words (the answer to part c.) using only S and/or U, how long would you have to make the words?

12. Make a tree diagram showing all arrangements of the letters in the word "REED". How many are there?

13. How many arrangements are there of the letters in the word "SEVEN"? In how many of these are the *E's* together? In how many are the *E's* separated?

14. Two players compete repeatedly until one wins two successive games or three games total. How many different sequences are possible? What if you change "three games total" to "four"?

15. A set of five regular octahedral dice have eight faces numbered 1, 2, 3, 4, 6, 8, 12, and 20. In how many different ways can these five dice fall? In how many ways will the five dice show a sum of six? of seven? What are the chances of rolling a sum of six or seven?

16. Not long ago, there were five teams in the East and Central and four in the West Division of both the American and National Baseball Leagues. How many different standings were possible for the 28 teams when ranked within their divisions? within their leagues? all together? Assume that there are no ties. How many World Series team match-ups were possible?

17. As I was going to St. Ives, I met a man with 7 wives. Each wife had 7 sacks, each sack had 7 cats, each cat had 7 kittens. Kittens, cats, sacks, wives ... how many were going to St. Ives?

A Tree Diagram for Arranging 4 Letters

How would you place the letters A, B, C, and D into these boxes to show all the ways to arrange these four letters? How many ways are there altogether to arrange these four letters? Can you see how to get that number using the multiplication rule?