

Homework 1 — Adjacent Transpositions

Due on Wednesday, September 10

Here are some definitions:

Web diagram: A diagram of horizontal and vertical lines which generates a permutation

Inversion: A pair of elements which is out of order compared to the “home” position

Here is some notation:

Let π be a permutation of the elements $\{1, 2, \dots, n\}$

$t(\pi)$ The number of inversions in π

$\text{parity}(\pi)$ The parity of $t(\pi)$, that is, whether $t(\pi)$ is odd or even

$l(\pi)$ The least number of lines needed in a web diagram to obtain π

Question 1:

Prove that $t(\pi)$ and $l(\pi)$ always have the same parity

Question 2:

Suppose the horizontal lines in a web diagram for π were allowed to connect only adjacent pairs of vertical lines. Find a formula, in terms of the notation given above, for the minimum number of lines needed to generate a given permutation π .

Question 3:

Prove that fewer lines will not suffice

Question 4:

Prove that you can always succeed with no more than that many lines