

## Bijection between Q and N

-3/3	-2/3	-1/3	0/3	1/3	2/3	3/3
-3/2	-2/2	-1/2	0/2	1/2	2/2	3/2
-3/1	-2/1	-1/1	0/1	1/1	2/1	3/1
-3/0	-2/0	-1/0	0/0	1/0	2/0	3/0
-3/-1	-2/-1	-1/-1	0/-1	1/-1	2/-1	3/-1
-3/-2	-2/-2	-1/-2	0/-2	1/-2	2/-2	3/-2

## Bijection between Q and N

- P We discard all fractions with "0" in the denominator
- P We discard all fractions not in lowest terms (to get rid of repeated fractions which are equal to each other)
- P We discard all fractions with a negative denominator (to get rid of all repeats of the form  $3/5 = -3/-5$ , or  $-2/7 = 2/-7$ )
- P What's left is a bijection between the natural numbers and the rationals, showing that they have the same cardinality

## Bijection Between N and R

P It turns out there is no such bijection!

P Proof: Suppose there was a bijection

- ▶ Then we would be able to create a list of all the real numbers that looked like this:

.374891347891238192...	.274891347891238192...
19.033301020030117481...	19.023301020030117481...
23.129873419873943714...	23.128873419873943714...
.00000013412844917...	.000100013412844917...
4.173894173841178434...	4.173884173841178434...
-2.718347193471471934...	-2.718348193471471934...
44.371839471394389471...	44.371839571394389471...
Etc...	Etc...

This number is not on the list → .2281885...

## Countable and Uncountable

- P A set is called *countable* if it is
  - ▶ Finite, or
  - ▶ Has the same cardinality as the natural numbers N
- P A set is called *uncountable* if it is not countable
- P Sets which are countable and infinite are called *countably infinite*
- P Note that countable sets are exactly those sets that you can make a list of.
- P We've seen two different sizes of infinity. It turns out that there are many more!
  - ▶ How many?

## Countable or Uncountable?

P The number of finite bit strings

▶ Can you make a list of all of them?

▶ Yes!

- 0
- 1
- 00
- 01
- 10
- 11
- 000
- 001
- 010
- 001
- 100
- ...

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## Countable or Uncountable?

P The set of all infinite bit strings

▶ Can you make a list of all of them?

▶ No!

▶ Suppose you could:

- 0100010101010101101010...
  - 1010010101010100100001...
  - 0101001010010101001010...
  - 111011111111101001111...
  - 00000000000000000000...
  - 11111011111111111111...
  - 1010100101111010000000...
  - ...
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- 1110100...

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