Some Homework Review

- Prove that $(A \cap B) \subseteq A$
  - We wish to prove that if $x \in (A \cap B)$, then $x \in A$.
  - To do this, note that if $x \in (A \cap B)$, then $x \in A$ and $x \in B$. In particular, $x \in A$.
- Prove that $A \subseteq (A \cup B)$
  - We wish to prove that if $x \in A$, then $x \in (A \cup B)$.
  - To do this, simply note that if $x \in A$, then $x \in (A \cup \textit{anything})$, in particular, $x \in (A \cup B)$.
- Prove that $A \cap (B - A) = \varnothing$
  - Consider an element x in $A \cap (B - A)$. Then $x \in A$ and $x \in B - A$. And if $x \in B - A$, then $x \in B$ and $x \notin A$.
  - But no x can both be in A and not be in A, so no such x exists, and $A \cap (B - A) = \varnothing$.

Membership Tables

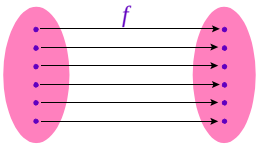
- We can prove that $A \cap (B - A) = \varnothing$ with membership tables as well (though we did not do this in class)

A	B	$B - A$	$A \cap (B - A)$
1	1	0	0
1	0	0	0
0	1	1	0
0	0	0	0

Since all entries in the $A \cap (B - A)$ column are 0, it means that no elements can be elements of $A \cap (B - A)$, which makes it empty.

Bijections

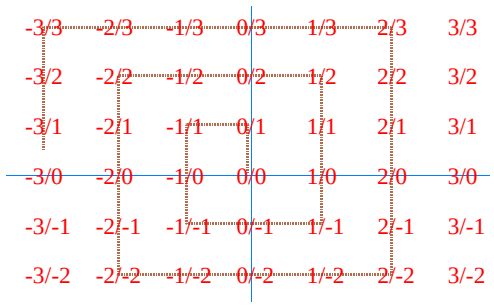
- A *bijection* is a function which is 1-1 and onto
- Two sets are said to have the same *cardinality* if there is a bijection between them
  - Note: For finite sets this says nothing new. The cardinality of a finite set is simply the number of elements it has. But for infinite sets, it does say something new!



Bijection

- For finite sets, you cannot have a bijection between two sets unless the sets have the same cardinality (number of elements)
- For infinite sets, the story is less intuitive:
  - Is there a bijection between the integers and the even integers: {...,-2 ,-1, 0, 1, 2, ...}↔{..., -4, -2, 0, 2, 4, ...}
    - $f(x) = 2x$
  - ...between {0, 1, 2, 3, ...} and {1, 2, 3, ...}
    - $f(x) = x + 1$
  - ...between Q and $\mathbb{N}$ = {0, 1, 2, 3, ...}.
    - Q is the set of all rational numbers, that is, numbers of the form a/b, where a and b are integers, and $b \neq 0$.

Bijection between Q and ℕ



Bijection between Q and ℕ

- We discard all fractions with "0" in the denominator
- We discard all fractions not in lowest terms (to get rid of repeated fractions which are equal to each other)
- We discard all fractions with a negative denominator (to get rid of all repeats of the form 3/5 = -3/-5, or -2/7 = 2/-7)
- What's left is a bijection between the natural numbers and the rationals, showing that they have the same cardinality