Predicates

- Expressions that have variables in them are not propositions
  - $x = 21$
  - $\alpha > \beta$
  - The graph *G* is Eulerian, but not Hamiltonian
- Such expressions are called *predicates*, or *propositional functions*
- We could let $P(x)$ be the expression: $x = 21$
  - Then $P(5)$ is false
  - But $P(21)$ is true

Universe of Discourse

- The *universe of discourse* is the set of all values that the variables in a predicate can take on
- It must be made clear what the universe of discourse is, for every variable involved, before one can do any work with a predicate
- Let $P(x, y)$ be the assertion that "*x* has eaten *y*"
  - *x* could be all ECU students and *y* all breakfast cereals
  - *x* could be all bears at the zoo and *y* could be all species of saltwater fish
  - As you see, the universe of discourse is important
- $P(x)$: *x* has a square root
  - Is $P(4)$ True? Is $P(7)$ True? Is $P(-2)$ true?

Quantifiers

- There are two kinds of quantifiers:
  - The *existential quantifier* $\exists$
  - The *universal quantifier* $\forall$
- Let $P(x)$ be some predicate:
  - The existential quantification $\exists x P(x)$ asserts that $P(x)$ is true *for some* value *x* in the universe of discourse
  - The universal quantification $\forall x P(x)$ asserts that $P(x)$ is true *for all* values *x* in the universe of discourse
- Let $P(x)$: *x* is the sum of the squares of two integers, where the univ. of disc. is all integers
  - Which of the following are true: $\exists x P(x)$, $\forall x P(x)$

Quantifying on More than one Variable

- Let $P(x, y)$ be the assertion that $xy = 1$, where the universe of discourse for each variable is the set of rational numbers
- Which of the following expressions are true:
  - $\forall x\ \exists y\ P(x, y)$
  - $\exists x\ \forall y\ P(x, y)$
  - $\exists x\ \exists y\ P(x, y)$
  - $\forall x\ \forall y\ P(x, y)$
  - $\forall x, (x \neq 0 \rightarrow \exists y P(x, y))$
- Suppose $P(x, y)$ is the assertion that $x \geq y$, where the universe of discourse is $\{1, 2, 3, 4, 5, 6, 7\}$.

Predicates become Propositions

- Notice that predicates become propositions when its variables are quantified
- But we must quantify *all* variables in the predicate for it to become a proposition

Sets

- Sets are collections of objects.
- The objects in a set are called its *elements*. If a is an element of set A, we write $a \in A$
- A set can be described in two ways:
  - Explicitly, by listing its elements within curly braces:
    - {a, b, c, d, e}
    - {Raleigh, Albany, Austin, ..., Sacramento}
    - {1, 2, 3, Albert Einstein, North Pole, 4, 5}
  - With *set builder* notation
    - $\{x \mid x > 10\}$
    - {p | p is prime and p is one more than a multiple of 4}
    - {a | a is a subset of {1, 2, 3} }
  - Note the need for a universe of discourse when using set builder notation

The Empty Set

- There is a unique set which contains no elements
- It's called the *empty set*
- It is denoted by "$\varnothing$"
- ...or "{}"

- It is a deceptively important set. Don't forget about it!
- It's the set analogue of the number "0"

Cardinality of a Set

- The *cardinality* of a set is simply the number of elements of that set
- For a finite set with n elements, the cardinality of that set is n
- If a set does not have a finite number of elements, then we say it is of infinite cardinality
  - We will see later that some infinities are bigger than others
- We denote the cardinality of a set S by $|S|$
- What is $|\varnothing|$?
  - 0

Equality of Sets

- Two sets are equal if they have exactly the same elements
- Note that this implies that if two sets are equal, then the have the same cardinality
- Thus, the following pairs are not equal:
  - $\{1, \{2\}\} \neq \{\{1\}, 2\}$
  - $2 \neq \{2\}$
  - $\{x \mid x^2 = 4\} \neq \{2\}$

Subsets

- If A and B are two sets, then we call A a *subset* of B if every element of A is also an element of B
  - $A \subseteq B \leftrightarrow (x \in A \rightarrow x \in B)$
- We write $A \subseteq B$
- If $A \subseteq B$, but $A \neq B$, then we can write $A \subset B$ to denote that A is a *proper subset* of B

Let $S = \{1, 2, 3\}$ $T = \{1, 2, 3, 4\}$
$U = \{0, 1, 2, 3\}$ $V = \{y \mid y \text{ is an integer} < 4\}$

Which of the sets above are subsets of which other sets? Can you suggest a nice way to picture these subset relationships?

The Power Set

The *power set* of a set A *is* the set of all subsets of A

- The power set is a set all of whose elements are also sets
- Find the power set of each of the following:
  - $\{1\}$
  - $\{a, b\}$
  - $\{\alpha, \beta, \gamma\}$
  - $\varnothing$