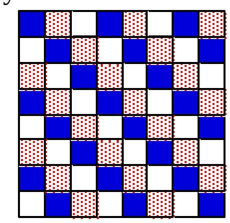
Some Proofs

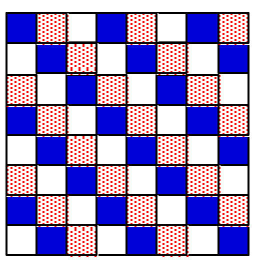
- Suppose $a > 0$ and $ab > 0$. Prove that $b > 0$.
- Proof by contradiction:
  - Suppose that b Ý 0. Then either $b = 0$ or $b < 0$.
    - if $b = 0$, then we would have $ab = 0$, which is false
    - if $b < 0$, then we would have $ab < 0$, which is also false
  - In either case, we get a contradiction, thus $b > 0$
- Note the use of *cases* in that proof

A Tiling Proof

- Prove that you cannot tile an 8×8 checkerboard with 1×3 trominos
  - Proof: Suppose you could. Then 64 would be a multiple of 3, which is false. (Proof by contradiction)
- Now prove that if you remove a single corner from the 8×8 checkerboard, then you cannot tile what remains with 1×3 trominos



- Hint: What if we color the checkerboard as shown to the right?
- There are 21 red, 21 white and 22 blue squares.



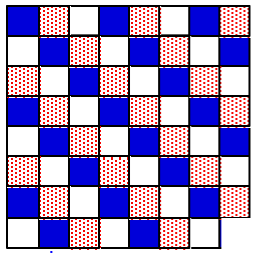
- We can see that if we remove a white square, then what remains cannot be tiled with 1×3 trominos
  - Because then there will be 20 white, 21 red and 22 blue squares
  - And if the region could be tiled, there would have to be the same number of each color of square
- Same for a red square
- But what if we remove a blue square? Then the resulting region *does* have an equal number of each.
- Does that mean the region *can* be tiled?
- No!

Converse versus Contrapositive

- We have the following theorem:
  - If a region of the checkerboard can be tiled with 1×3 trominos, then that a 3-coloring of that region must have the same number of each color
  - But the *converse* is not necessarily true:
    - Namely, that if a region has the same number of each color, then it can necessarily be tiled
  - In general, converses *might* be true, but you can't count on it
  - The contrapositive *is* always true, though.
    - If a 3-coloring does not have an equal number of each color, then the region cannot be tiled

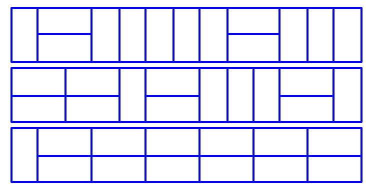
Dealing with the blue corner

- So, how do we prove the region with a blue corner removed cannot be tiled?
- Suppose it could!
  - Where do you think we go from here?
- Then we could rotate that tiling clockwise 90 degrees to obtain a tiling with the white corner removed...
- Clearly a contradiction.



Dominizing a 2×*n* rectangle

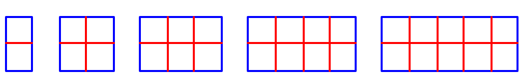
- How many different ways are there to dominize a 2×13 rectangle with 1×2 dominos?
- Here are some sample dominizations:



Can you discover a strategy for getting this answer?

A Problem-Solving Strategy

- This is not a proof-technique
- Sometimes, when you have to solve a hard problem, it makes sense to try to *solve a simpler problem*
  - How would you change the problem to make it simpler?
    - Try a different size grid...
    - Try *many* different sized grids, and see if you can find a pattern!



ways: 1 2 3 5 8

Proving a Conjecture

- It looks like each number in the sequence is the sum of the two previous numbers: $f(n) = f(n-1)+f(n-2)$
- How could we *prove* that this pattern will continue?
  - There are 2 ways to start dominizing a 2×*n* grid:
    - Lay the first domino vertically
    - In which case there are $f(n-1)$ ways to finish the job
    - Lay the first domino horizontally
    - In which case the one above or below it must also lie horizontally, and then there are $f(n-2)$ ways to finish the job
  - Since those are all the cases, we've proven that $f(n) = f(n-1)+f(n-2)$

n	# ways
1	1
2	2
3	3
4	5
5	8
6	13
7	21
8	34
9	55
10	89