Integers

- These are the integers: {..., –3, –2, –1, 0, 1, 2, ...}
- These are the natural numbers: {0, 1, 2, ...}
- We will be concerned at first with the divisibility properties of the integers and natural numbers

Multiple

- 10 is a multiple of 2
- 21 is a multiple of 7
- 100 is a multiple of 10
- 42 is a multiple of –6
- –90 is a multiple of 15
- –20 is a multiple of –4

- In general, given two integers *a* and *b*, we say *b* is a *multiple* of *a* if $b = ma$ for some integer *m*.

Divides

4 divides 20	4 | 20
10 divides 100	10 | 100
–9 divides 81	–9 | 81
6 divides –30	6 | –30
1 divides 7	1 | 7
–1 divides –8	–1 | –8

- In general, we say that *a divides b* if *b* is a multiple of *a*. This is written: $a \mid b$

Factors and Divisor

- If *a* and *b* are integers, and *a* divides *b*, then we call *a* a *factor* of *b*
- "divisor" and "factor" mean the same thing
- Can you list all factors of 10?
  - –10, –5, –2, –1, 1, 2, 5, 10
- Can you list all positive factors of 20?
  - 1, 2, 4, 5, 10, 20
  - Can you see why you need only the first half of that list?
  - Will there always be an even number of factors?
- Can you list all the positive factors of 36?
  - 1, 2, 3, 4, 6, 9, 12, 18, 36

Prime Numbers

- An integer p greater than 1 is called *prime* if its only positive divisors are 1 and p
- Which of the following are prime:
  - 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17
- A *prime factor* is a factor which is prime
  - What are the prime factors of 60?
  - 2, 3 and 5
- The *prime factorization* of an integer n is an expression of n as the product of primes
  - $100 = 2 \cdot 2 \cdot 5 \cdot 5$
  - $99 = 3 \cdot 3 \cdot 11$

Greatest Common Divisor

- Reduce the fraction: 30/100
- Reduce the fraction: 18/33
- Given two integers a and b, we call the greatest divisor common to both a and b their *greatest common divisor*, denoted gcd(a, b).
- gcd(30, 100) = 10
- gcd(18, 33) = 3
- gcd(20, 60) =
  - 20
- gcd(105, 154) =
  - 7

Prime Factorization and gcd

- We can easily find the gcd of two integers given their prime factorizations:
  - $a = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 11 \cdot 19 \cdot 41 \cdot 47$ (= 604114500)
  - $b = 2 \cdot 5 \cdot 5 \cdot 11 \cdot 41 \cdot 101 \cdot 103$ (= 234587650)
- We consider the greatest amount of each prime factor that they have in common
- The product of these prime factors gives the greatest common divisor
  - $gcd = 2 \cdot 5 \cdot 5 \cdot 11 \cdot 41$ (= 22550)
- This gives us an algorithm for determining the gcd of two integers, assuming we can find prime factorizations.

Our algorithm for finding gcd

- Find gcd(19841693512938, 189341078342178)
  - Do you want to find the prime factorizations of those two numbers?
  - ``(2)*``(3)*``(11)*``(41)*``(42139)*``(174007)
  - ``(2)*``(3)*``(7)*``(23)*``(103)*``(1902963661)
- How would you decide that 1,902,963,661 is prime?
  - Can you come up with an algorithm for deciding this?
  - Would your algorithm work on 138941728941783279897418934189237819274184319 7?
- Thus, our gcd algorithm is not very good. We will find a better one on Thursday

Relatively Prime

- Two integers *a* and *b* are called *relatively prime* if gcd(*a*, *b*) = 1
- That is, they are relatively prime if they have no non-trivial common divisor
- Which integers from 1 to 10 are rel. prime to 10?
  - 1, 3, 7, 9
- We define the *phi-function* $N(n)$ to be the number of integers in the range {1, 2, ..., *n*} which are relatively prime to *n*
  - Everyone: Find $N(30)$