Computer Science 4602
Fall 2020
Quiz 4

Answer all of the questions. Clearly indicate the best answer to each multiple-choice problem (marked [MC]), even if no answer is ideal.

Read each question carefully. Be sure that you understand what the question is asking before you answer the question. Some of the questions are designed to see whether you understand definitions well enough to apply them.

Check your answers with a critical eye.

1. [MC] Function f is a mapping reduction from A to B provided
   (a) f is computable and for all x, $f(x) \in B \iff f(x) \in A$.
   (b) f is computable and for all x, $x \in B \iff f(x) \in A$.
   (c) f is computable and for all x, $x \in A \iff f(x) \in B$.
   (d) f is computable and for all x, $f(x) \in A \iff f(x) \in B$.

2. [MC] We say that $A \leq_m B$ if
   (a) A is computable but B is not computable.
   (b) B is computable but A is not computable.
   (c) there exists a mapping reduction from B to A.
   (d) there exists a mapping reduction from A to B.

3. [MC] One way to prove that a set A is uncomputable is to show that
   (a) $A \leq_m$ HLT where HLT is the halting problem.
   (b) $\overline{A} \leq_m$ HLT where HLT is the halting problem.
   (c) HLT $\leq_m A$ where HLT is the halting problem
   (d) A is nontrivial.
   (e) A is infinite.

4. [MC] Suppose that the alphabet includes symbol 0. Define

$$\begin{aligned} A &= \{p \mid \text{Run}(p,\ 0)\downarrow\} \\ \text{HLT} &= \{(p, x) \mid \text{Run}(p, x)\downarrow\} \end{aligned}$$

That is, A is the set of all programs that terminate when their input is 0, and HLT is the halting problem. Which of the following functions is a mapping reduction from A to HLT?

(a) $f(p, 0) = p$

(b) $f(p) = (p, 0)$

(c) $f(p, x) = p$

(d) $f(p) = p$

(e) $f(p) = 0$

5. Suppose that A and B are computable languages where B is nontrivial. (That is, $B \neq \{\}$ and $\overline{B} \neq \{\}$.) Assume that program p_A computes A and program p_B computes B. Show that $A \leq_m B$ by giving a mapping reduction from A to B.

6. If p is a program, $L(p)$ is defined to be the set $\{x \mid \text{Run}(p, x) \cong 1\}$. That is, $L(p)$ is the set of strings on which p answers 1. Let $D = \{p \mid L(p)$ is a finite set$\}$.

   (a) Read the definition of D carefully. Is D a finite set?

   (b) Is D computable? Justify your answer.

7. A program can produce a number as its result by producing a string such as “400”. For this exercise, let’s restrict attention to programs that take an integer and produce an integer.

   Suppose $A = \{p \mid \mathrm{Run}(p, 2) = 0\}$ and $B = \{p \mid \mathrm{Run}(p, 2) = 1\}$. Give a mapping reduction from A to B. Write down the property the reduction needs to have, specific to this reduction, before you define the reduction function that has that property.