You have 50 minutes. Answer all of the questions on the exam. Circle the letter of the best answer to each multiple-choice problem (marked [MC]), even if no answer is ideal. You may use one prepared 8.5×11 sheet of paper during the exam. **Check your work.**

1. [MC] Function \( f \) is a mapping reduction from \( A \) to \( B \) provided
   
   (a) \( f \) is computable and for all \( x, f(x) \in A \iff f(x) \in B \).
   
   (b) \( f \) is computable and for all \( x, f(x) \in B \iff f(x) \in A \).
   
   (c) \( f \) is computable and for all \( x, x \in B \iff f(x) \in A \).
   
   (d) \( f \) is computable and for all \( x, x \in A \iff f(x) \in B \).

2. [MC] We say that \( A \leq_m B \) if
   
   (a) there exists a mapping reduction from \( B \) to \( A \).
   
   (b) there exists a mapping reduction from \( A \) to \( B \).
   
   (c) \( A \) is computable but \( B \) is not computable.
   
   (d) \( B \) is computable but \( A \) is not computable.

3. [MC] One way to prove that a set \( A \) is uncomputable is to show that
   
   (a) \( H \leq_m A \) where \( H \) is the halting problem.
   
   (b) \( A \leq_m H \) where \( H \) is the halting problem.
   
   (c) \( A \) is partially computable.
   
   (d) the complement of \( A \) is partially computable.
4. [MC] Suppose that the alphabet includes symbol 0. Define

\[
A = \{p \mid \phi_p(0) \downarrow\}
\]

\[
H = \{(p, x) \mid \phi_p(x) \downarrow\}
\]

That is, \(A\) is the set of all programs that terminate when their input is 0, and \(H\) is the halting problem. Which of the following functions is a mapping reduction from \(A\) to \(H\)?

(a) \(f(p, 0) = p\)
(b) \(f(p) = (p, 0)\)
(c) \(f(p, x) = p\)
(d) \(f(p) = p\)
(e) \(f(p) = 0\)

5. Suppose that \(A\) and \(B\) are computable languages where \(B\) is nontrivial. (That is, \(B \neq \{\}\) and \(\overline{B} \neq \{\}\).) Show that \(A \leq_m B\) by giving a mapping reduction from \(A\) to \(B\).
6. Let $D = \{p \mid \phi_p(0) \uparrow\}$. Show that $D$ is uncomputable.
7. Artificial neural networks are one approach to solving computational problems whose algorithms are difficult to derive by hand. The idea is that a computer solves a problem by simulating a network of artificial neurons.

As our understanding of artificial neural networks improves and computers for simulating neurons become more powerful, can we expect the halting problem to become solvable using artificial neural networks? Why or why not?
8. A program can produce a string or a number as its result. For this exercise, let’s restrict attention to programs that take an integer and produce an integer.

Suppose $A = \{p \mid \phi_p(0) = 0\}$ and $B = \{p \mid \phi_p(0) = 1\}$. Give a mapping reduction from $A$ to $B$. Be sure that you know what properties the reduction needs to have before you start to describe the reduction.