Write clear, readable answers for each of the following. **You will be graded down for sloppy work and for work that is difficult to understand.**

1. Write a clearly legible T to the left of each of the following and a clearly legible F to the left of each that is false.

   (a) A maximum matching is always a complete matching.
   (b) A complete matching is always a maximum matching.
   (c) $K_{2,2}$ is isomorphic to $C_4$.
   (d) $K_{2,3}$ is planar.
   (e) There are only two nonplanar graphs, namely $K_5$ and $K_{3,3}$.
   (f) $Q_3$ is bipartite.
   (g) $Q_2$ is bipartite.
   (h) All planar graphs are 3-colorable.
   (i) All planar graphs are 4-colorable.

2. Give an isomorphism from $G$ to $H$, where $G$ and $H$ are as follows.

   ![Graphs G and H](image-url)
3. Show the subgraph of graph $G$ from question 2 induced by set of vertices $\{1, 2, 3\}$.

4. Show a subgraph of graph $G$ from question 2 that is not an induced subgraph.

5. Explain why $K_{1,3}$ and $C_4$ are not isomorphic.

6. Suppose that a simple graph $G$ has 90 vertices and 250 edges. What is the sum of the degrees of all of the vertices?

   **Answer:**

7. How many connected components does a complete matching of $K_{3,3}$ have?

   **Answer:**
8. Suppose that $G$, $H$ and $I$ are three simple graphs. Suppose that $f$ is an isomorphism from $G$ to $H$ and $g$ is an isomorphism from $H$ to $I$. Give an isomorphism from $G$ to $I$. Prove that it is an isomorphism from $G$ to $I$. 
9. Suppose that there are five young women and six young men on an island. Each woman is willing to marry some of the men on the island and each man is willing to marry any woman who is willing to marry him.

Anna is willing to marry Jason, Larry and Matt. Barbara is willing to marry Kevin and Larry. Carol is willing to marry Jason, Nick and Oscar. Diane is willing to marry Jason, Larry, Nick and Oscar. Elizabeth is willing to marry Jason and Matt.

(a) Draw a diagram of a bipartite graph that models all of the possible marriages. Label each vertex by the first letter of the name of the person modeled by that vertex.

(b) Draw a diagram of a matching such that each woman is matched with a man whom she is willing to marry.