Computer Science 2400
Fall 2021
Practice Quiz 4b
Elementary Number Theory

Circle the letter of the best answer for each multiple-choice question. Use a calculator where it helps.

1. What is 312 mod 7?

2. What is the prime factorization of 28?

3. Suppose $x \equiv 45 \pmod{11}$. What is x^{65} mod 11?

4. What is 5^{90} mod 91? (Hint. 91 is a prime number.)

5. What is 45^{32} mod 7?

6. What time does a 24 hour clock read 81 hours after it reads 1:00?

7. What are the quotient q and remainder r when you divide 104 by 7, according to the division algorithm (division theorem)?

8. What are the quotient q and remainder r when you divide -104 by 7, according to the division algorithm? (Hint. Check the requirements of the division algorithm.)

9. What is the result of converting 10010_2 from binary to decimal?

10. What is the result of converting 59 from decimal to binary?

11. What is gcd(114, 304)?

12. What is lcm(114, 304)?

13. Positive integers x and y are relatively prime if and only if

    (a) $\gcd(x, y) = 1$.
    (b) x and y are both prime.
    (c) $\operatorname{lcm}(x, y) = x$.

14. The inverse of 16 (mod 31) is 2. Solve congruence

    $$16x + 2 \equiv 21 \pmod{31}$$

    for x. Express the answer as a number from 0 to 30.

15. Suppose you select a random integer in the range from 2 to 1,000,000,000, with each integer in that range equally likely. Which of the following is closest to the probability that the selected integer is prime? (Hint. Approximately what fraction of the numbers from 2 to n are prime? Use the prime number theorem and a calculator. Divide the number of primes from 2 to n by n to get the probability that a random number from 1 to n is prime.)

    (a) 0.1

(b) 0.05

(c) 0.01

(d) 0.005