Write clear and concise proofs.

1. Prove: There is a positive integer \( x \) that is equal to the sum of all of the positive integers that are less than \( x \).

2. Prove: For every integer \( x \), there is an integer \( y \) such that \( y + 3 = x \).
3. Prove: If $x$ is a real number and $x \leq 3$ then $12 - 7x + x^2 \geq 0$. (Hint. For which values of $x$ is $12 - 7x + x^2 = 0$? Sketch a graph of $y = 12 - 7x + x^2$.)

4. What would be the starting point in a proof by contrapositive of: If $x < 0$ and $xy > 0$ then $y < 0$. That is, what is the contrapositive of the goal?
5. Prove by contradiction: If a group of 9 kids have won a total of 100 trophies, then at least one of the 9 kids has won at least 12 trophies.

6. A real number $x$ is rational if there exist integers $a$ and $b$ where $b \neq 0$ such that $x = a/b$. Prove using the contrapositive: For every pair of real numbers $x$ and $y$, if $x$ is rational and is $xy$ not rational, then $y$ is not rational. (Hint. Write what you are asked to prove in logic. Write the contrapositive of that in logic. Remember that $(\neg p \lor q) \equiv (p \to q)$. Also, $(p \to (q \to r)) \equiv ((p \land q) \to r)$.)