**Computer Science 2400**
**Fall 2021**
**Practice Quiz 2b**
**Practical Proofs**

Write clear and concise proofs.

1. Prove: There is a positive integer $x$ that is equal to the sum of all of the positive integers that are less than $x$.

2. Prove: For every integer $x$, there is an integer $y$ such that $y + 3 = x$.

3. Prove: If $x$ is a real number and $x \leq 3$ then $12-7x+x^2 \geq 0$. (Hint. For which values of $x$ is $12-7x+x^2 = 0$? Sketch a graph of $y = 12-7x+x^2$.)

4. What would be the starting point in a proof by contrapositive of: If $x < 0$ and $xy > 0$ then $y < 0$. That is, what is the contrapositive of the goal?

5. Prove by contradiction: If a group of 9 kids have won a total of 100 trophies, then at least one of the 9 kids has won at least 12 trophies.

6. A real number $x$ is rational if there exist integers $a$ and $b$ where $b \neq 0$ such that $x = a/b$. Prove using the contrapositive: For every pair of real numbers $x$ and $y$, if $x$ is rational and is $xy$ not rational, then $y$ is not rational. (Hint. Write what you are asked to prove in logic. Write the contrapositive of that in logic. Remember that $(\neg p \vee q) \equiv (p \rightarrow q)$. Also, $(p \rightarrow (q \rightarrow r)) \equiv ((p \wedge q) \rightarrow r)$.)