Circle the letter of the best answer for each multiple-choice question.

1. Write a clearly legible T to the left of each of the following that is true and a clearly legible F to the left of each that is false. The domain of discourse is the set of all integers.

   (a) \( \exists x (x + x = 1) \)
   (b) \( \exists x (x + 2 = 1) \)
   (c) \( \forall x (x^2 - x \neq 1) \)
   (d) \( \forall x (x^2 - x \neq 0) \)
   (e) \( \forall x (x^2 > 0) \)
   (f) \( \exists x (x^2 > 0) \)
   (g) \( \forall x \forall y (xy = 1) \)
   (h) \( \exists x \exists y (xy = 1) \)
   (i) \( \forall x \exists y (xy = 1) \)
   (j) \( \exists x \forall y (xy = 1) \)
   (k) \( \exists x \exists y ((x + y = x \land y \neq 0) \)
   (l) \( \forall x \forall y ((x + y = x \land y \neq 0) \)
   (m) \( \exists x \forall y (xy = y) \)

2. Which of the following is equivalent to \( \neg \exists x (P(x) \lor Q(x)) \)

   (a) \( \forall x (\neg P(x) \land \neg Q(x)) \)
   (b) \( \forall x (\neg P(x) \lor \neg Q(x)) \)
   (c) \( \exists x (\neg P(x) \land \neg Q(x)) \)
   (d) \( \exists x (\neg P(x) \lor \neg Q(x)) \)
3. Suppose
   \( P(x) \) means “\( x \) showed up with a pencil,”
   \( C(x) \) means “\( x \) showed up with a calculator.”

Express each of the following in first-order logic using predicates \( P(x) \) and \( C(x) \).

(a) At least one of the students showed up with a pencil.

(b) Every student showed up with a pencil or a calculator (or both).

(c) Every student who showed up with a calculator also had a pencil.

(d) There is a student who showed up with both a pencil and a calculator.

(e) At least two different students showed up with a pencil.

(f) Exactly one student showed up with a pencil.