M_1 $k=1$ M_n

$$\left([3\times 4] \,|\, [4\times 9][9\times 20][20\times 6][6\times 12][12\times 7][7\times 2][2\times 9] \right)$$

Product (i, j) // find the least # of multiplications.

```
if i=j return 0
min = ∞
for k = i to j-1
    m = Product(i,k) + Product(k+1,j) + p_i q_k q_j
    if m < min
        min = m
return m
```

M_i = i^{th} matrix

$P_{ij} = M_i \cdot M_{i+1} \cdot \cdots \cdot M_j$

$i \leq j$.

Problem: Find P_{1n} in as few multiplications as possible

Let M_i have dimensions $p_i \times q_i$

```
Product (i, j)  // find the least # of
                   multiplications.
 if i = j return 0
 if memo(i, j) exists
      return memo(i, j)
 min = ∞
 for k = i to j-1
     m = Product(i, k) + Product(k+1, j) + p_i q_k q_j

     if m < min
          min = m

 memo(i, j) = m
 return m
```

[3x4] [4x9] [9x20] [20x6] [6x12] [12x7] [7x2] [2x9]

0 0 0 0 0 0 0 0 O(1) each

108 720 1080 1440 504 168 126 O(1) each

O(2) time

$\boxed{648}$ $\boxed{1200}$

$\boxed{1008}$ O(3) time

$108 + 3 \cdot 9 \cdot 20 = 648$

$720 + 3 \cdot 4 \cdot 20 = \text{more}$

$720 + 4 \cdot 20 \cdot 6 = 1200$

$1080 + 4 \cdot 9 \cdot 6 = 1296$

O(k)

O(n-1) time.

On the exam, I would like you to be able to complete this red array, but with smaller #'s.

$(0)(1200) = 0 + 1200 + 3 \cdot 4 \cdot 6 = 1272$

$(108)(1080) = 108 + 1080 + 3 \cdot 9 \cdot 6 = 1352$?

$(648)(0) = 648 + 0 + 3 \cdot 20 \cdot 6 = 1008$

$[2\times3][3\times4][4\times2][2\times3][3\times3]$ Practice.

Running time for n matrices = amount of time to fill in the memo array(s)

~~for the~~

Running time = n for the 0's

$1\cdot(n-1)$ for the 1^{st} row

$2\cdot(n-2)$ for the 2^{nd} row

$\vdots$

$(n-1)\cdot 1$ for the last row

$$n+\sum_{k=1}^{n-1} k(n-k)$$

$$= n+\sum kn-k^2$$

$$= n+n\sum k-\sum k^2$$

$$= n+n\frac{n(n-1)}{2} - \frac{(n-1)(n)(2n-1)}{6}$$

$$= O(n^3)$$