Thm: If $d|a$ and $d|b$, then $d | ax+by$ for any integers x, y.

Proof: (use the definition of $d|a$)

($d|a$ means a is a multiple of d)

$d|a \rightarrow a$ is a multiple of d

$\rightarrow a = d \cdot k$ for some $k \in \mathbb{Z}$.

$d|b \longrightarrow b = d \cdot l$ for some $l \in \mathbb{Z}$.

the conclusion we want is $d | ax+by$.

which means $ax+by = d \cdot (\text{some integer})$

So $ax + by = d \cdot k \cdot x + d \cdot l \cdot y = d(kx + ly)$

$\rightarrow d | ax+by$. ∎

Thm: $\sqrt{2}$ is irrational.

Def: x is rational if $x = \frac{a}{b}$ for some $a, b \in \mathbb{Z}$.

Proof: Suppose $\sqrt{2}$ is rational.

Then $\sqrt{2} = \frac{a}{b}$, $a, b \in \mathbb{Z}$. for some reduced fraction $\frac{a}{b}$.

So $b\sqrt{2} = a$

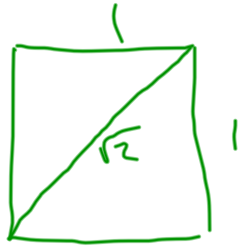
$\rightarrow b^2 \cdot 2 = a^2$

$\rightarrow a^2$ is even

$\rightarrow a$ is even

$\rightarrow a = 2 \cdot k$ for some $k \in \mathbb{Z}$.

$\rightarrow a^2 = (2k)^2 = 4k^2$



$2 \cdot b^2 = 4k^2 = a^2$

$\rightarrow b^2 = 2k^2$

$\rightarrow b^2$ is even

$\rightarrow b$ is even

So... a and b are both even.

Contradiction, $\lightning$ $\blacksquare$