

x	$2^x \pmod 7$
0	1
1	2
2	4
3	1
4	2
5	4
6	1
25	1
100	2

$2 \pmod 7 = 2$
 multiple of 3
 $2 \pmod 7 = 1$

x	$3^x \pmod 7$
0	1
1	3
2	2
3	6
4	4
5	5
6	1
7	3
8	2
9	6
10	4
11	5
12	1
13	3
14	2
15	6
16	4
17	5
18	1
19	3
20	2
21	6
22	4
23	5
24	1
25	3
26	2
27	6
28	4
29	5
30	1
31	3
32	2
33	6
34	4
35	5
36	1
37	3
38	2
39	6
40	4
41	5
42	1
43	3
44	2
45	6
46	4
47	5
48	1
49	3
50	2
51	6
52	4
53	5
54	1
55	3
56	2
57	6
58	4
59	5
60	1
61	3
62	2
63	6
64	4
65	5
66	1
67	3
68	2
69	6
70	4
71	5
72	1
73	3
74	2
75	6
76	4
77	5
78	1
79	3
80	2
81	6
82	4
83	5
84	1
85	3
86	2
87	6
88	4
89	5
90	1
91	3
92	2
93	6
94	4
95	5
96	1
97	3
98	2
99	6
100	4

$3^{100} \pmod 7 = 4$

Fermat's Little Theorem:

If p is prime and a is not a multiple of p , then $a^{p-1} \bmod p = 1$.

$$12^{16} \bmod 17 = 1$$

$$100^{16} \bmod 17 = 1$$

$$1806^{1600} \bmod 17 = 1$$

$$(806)^{16} \bmod 17 = 1$$

$$\begin{aligned} (1806^{16})^{100} \bmod 17 \\ = 1^{100} \bmod 17 \\ = 1 \end{aligned}$$

By F.L.T., $4000^{12} \pmod{13} = 1$

$$4000^{20000} \pmod{13}$$

$$13 \overline{) 4000} \begin{array}{r} 307 \\ \underline{390} \\ 100 \\ \underline{91} \\ 90 \\ \underline{78} \\ 12 \end{array} \text{ (R9)}$$

$$(4000^{12})^{1000} = 4000^{12000} \pmod{13} = 1$$

So what's 4000^{8000} ?

$$12 \overline{) 20000} \begin{array}{r} 1666 \\ \underline{1992} \\ 8 \end{array} \text{ (R8)}$$

$$4000^{12 \cdot 1666 + 8} = (4000^{12 \cdot 1666}) \cdot 4000^8$$

that's 1
that's what we need

Since every 12th power of 4000 gives a 1, we can divide 20000 by 12, and look at only the remainder

$$4000^8 \pmod{13} = 9^8 \pmod{13}$$

Reduce 4000 mod 13, first, then exponentiate.

$$9^8 \pmod{13}$$

$$9^2 \pmod{13} = 3$$

~~$$(9^2)^2 = 9^4 = (3)^2$$~~

$$9^4 = (9^2)^2 = 3^2 = 9 \pmod{13}$$

$$9^8 = (9^4)^2 = 9^2 = 3 \pmod{13}$$

multiples of 13

0

13

26

39

52

65

78

91

104

$$50000 \dots - - \left| \begin{array}{l} 20000 \\ 40000 \end{array} \right. \pmod{13} = 3$$

$2186^{19788} \pmod{11}$ exponent can be reduced mod 10

↑
base can be reduced mod 11

$$2186 \% 11 = 8$$

$$19788 \% 10 = 8$$

$$\begin{aligned} 8^8 \pmod{11} &= 64^4 = 9^4 = 81^2 \\ &= 4^2 = 16 \equiv 5 \pmod{11} \end{aligned}$$