x	$2^x \bmod 7$
0	1
1	2
2	4
3	1
4	2
5	4
6	1
99	1
100	2

$$2^{100} \bmod 7 = 2$$

$$2^{\text{multiple of } 3} \bmod 7 = 1$$

x	$3^x \bmod 7$
0	1
1	3
2	2
3	6
4	4
5	5
6	1
7	3
8	2
9	6
10	4
11	5
96	1
97	3
98	2
99	6
100	4

$$3^{100} \bmod 7 = 4$$

Fermat's Little Theorem:

If p is prime and a is not a multiple of p, then $a^{p-1} \bmod p = 1$.

$12^{16} \bmod 17 = 1$

$100^{16} \bmod 17 = 1$

$1806^{1600} \bmod 17 = 1$

$(1806)^{16} \bmod 17 = 1$

$(1806^{16})^{100} \bmod 17$

$= 1^{100} \bmod 17$

$= 1$

By F.L.T., $4000^{12} \bmod 13 = 1$

$4000^{20000} \bmod 13$

$$13 \overline{)4000} \quad 307 \ \text{R}9$$

$(4000^{12})^{1000} = 4000^{12000} \bmod 13 = 1$

So what's 4000^{8000}?

Since every 12^{th} power of 4000 gives a 1, we can divide 20000 by 12, and look at only the remainder

$$12 \overline{)20000} \quad 1666 \ \text{R}8$$

that's 1

that's what we need

$4000^{12 \cdot 1666 + 8} = (4000)^{12 \cdot 1666} \cdot 4000^8$

$4000^8 \bmod 13 = 9^8 \bmod 13$

Reduce 4000 mod 13, first. then exponentiate.

multiples of 13

0
13
26
39
52
65
78
91
104

$9^8 \bmod 13$

$9^2 \bmod 13 = 3$

$\cancel{(9^2)^2 = 9^4 = (3)^2}$

$9^4 = (9^2)^2 = 3^2 = 9 \bmod 13$

$9^8 = (9^4)^2 = 9^2 = 3 \bmod 13$

$50000 \ldots + \boxed{4000^{20000} \bmod 13 = 3}$

exponent can be reduced mod 10

$$2186^{19788} \bmod 11$$

base can be reduced mod 11

2186 % 11 = 8

19788 % 10 = 8

$$8^8 \bmod 11 = 64^4 = 9^4 = 81^2$$

$$= 4^2 = 16 = \boxed{5} \bmod 11$$