

What is the least # of rungs needed to generate the permutation 53241876

To find cycles, compare to home position:

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 3 & 2 & 4 & 1 & 8 & 7 & 6 \end{array}$$

$$\text{Cycle structure} = (1, 5)(2, 3)(4)(6, 8)(7)$$

$$\text{Least \# rungs needed} = n - c = 8 - 5 = 3$$

$$= 2 + 2 + 1 + 2 + 1$$

$$- (1 + 1 + 1 + 1 + 1)$$

$$(1, 2, 4, 6, 7)(3, 9, 5, 8)$$

$$\text{need } 9 - 2 = 7 \text{ rungs}$$

Is this puzzle solvable:

4 <sub>1</sub>	2 <sub>2</sub>	8 <sub>3</sub>
7 <sub>4</sub>	3 <sub>5</sub>	5 <sub>6</sub>
6 <sub>7</sub>	1 <sub>8</sub>	

1 2 3 4 5 6 7 8

4 2 8 7 3 5 6 1

$$3 + 1 + 5 + 4 + 1 + 1 + 1 + 0 = 16 \text{ inversions}$$

# inversions = 16, even, so the puzzle is solvable.

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Cycle structure =  $(1, 8, 3, 5, 6, 7, 4)(2)$

$$\# \text{ cycles} = 2$$

$$n - c = 8 - 2 = 6$$

This perm. can be achieved w/ 6 moves.  
Even. Solvable.

Even cycles (eg 2 3 4 5 6 7 8 1)

have odd parity. (8-cycle)

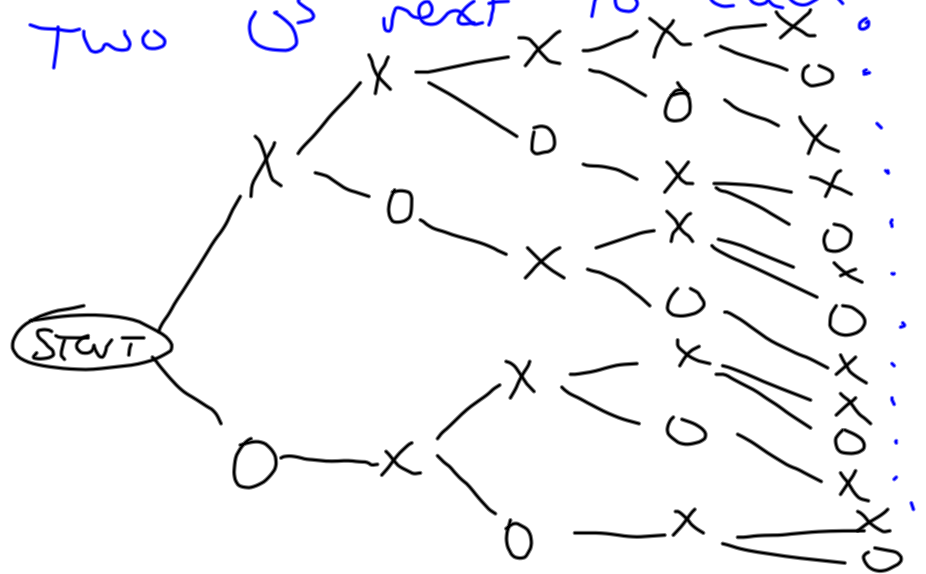
7 cycles

$$n - c = n - 1 \\ = 8 - 1 = 7.$$

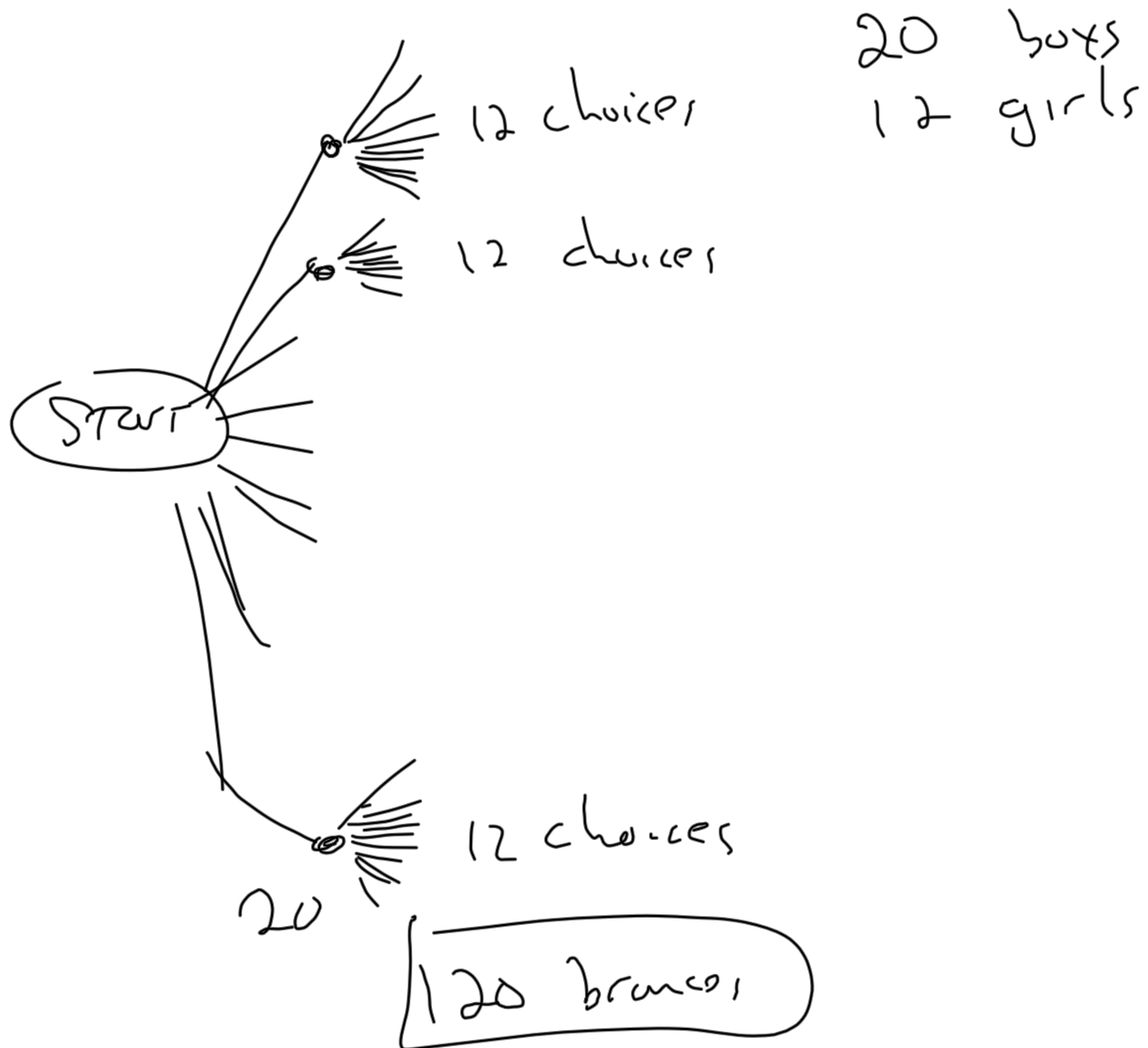
$n = \#$  elements

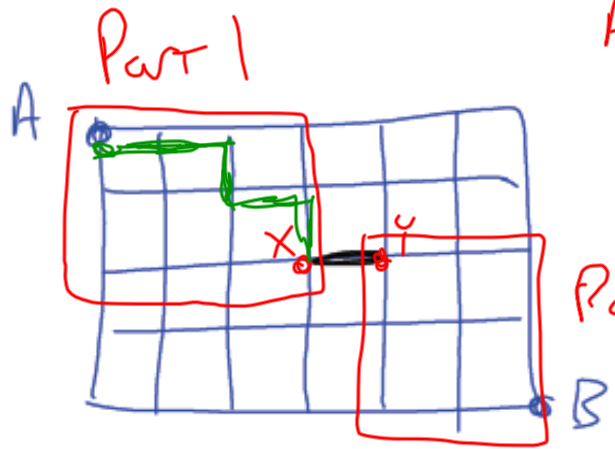
$c = \#$  cycles

# X-O sequences of length 5 without  
two 0s next to each other.



13 ways





$A \rightarrow X$  can be done in  $\frac{5!}{2!3!} = 10$  ways

SSEEE

$X \rightarrow Y$  can be done in  $\frac{4!}{2!2!} = 6$  ways

SSEEE

So, the total # of ways is 60

$= 10 \times 6$

$$\frac{5!}{2! \cdot 3!} = \frac{5 \cdot \overset{2}{\cancel{4}} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot 1 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 5 \cdot 2 = 10$$

Eric has 12 children, and wants to select ~~the~~ four to bring to the game.

# ways to make his choice? {express... integer.

$$\binom{12}{4} = \frac{12!}{4! \cdot 8!} = \frac{\cancel{12} \cdot \cancel{11} \cdot \overset{5}{\cancel{10}} \cdot \cancel{9} \cdot \cancel{8!}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1} \cdot \cancel{8!}} = \boxed{495}$$

Y Y Y Y N N N N N N N N