How many ways to select 4 from 10?

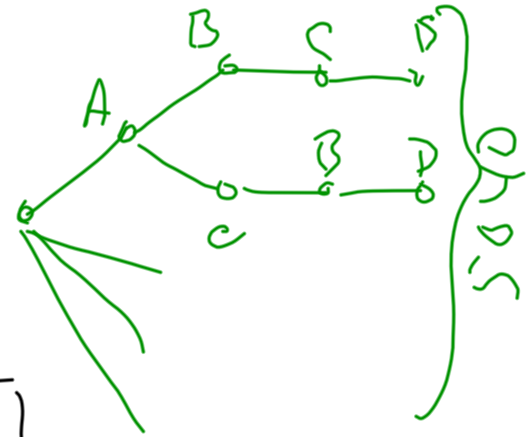
Correct count

$$\frac{10\times9\times8\times7}{4!}$$

$10\times9\times8\times7 = 5040$

$\underline{A}\ \underline{B}\ \underline{C}\ \underline{D}$

A C B D

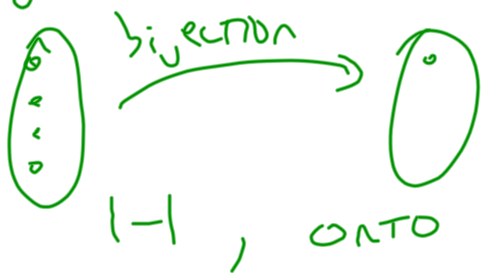


A B C D E F G H I J

Y N Y N Y Y N N N N $\longleftrightarrow$ {A, C, E, F}

N N Y N N Y N Y N Y $\longleftrightarrow$ {C, F, H, J}

There is a bijection between subsets of size 4 and anagrams of "YYYY NNNNNN"



$$\#\text{ subsets} = \#\text{anagrams} = \frac{10!}{4!\cdot 6!} = 210.$$

$$\frac{10!}{4!\cdot 6!} = \frac{10\cdot \cancel{9}^{3}\cdot \cancel{8}\cdot 7\cdot \cancel{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1\cdot \cancel{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}} = 210$$

Exam Friday Feb 22

Selecting — but not ordering.

<u>Def</u>: the # of ways to <u>select</u> k objects from n distinct objects is called "n choose k", and is written $\binom{n}{k}$ in this class, or as $_nC_k$ or $C(n,k)$ in other places.

Evaluate $\binom{7}{3} = \frac{7!}{3! \cdot 4!} = 35$

ABCDEFG
YYYNNNN

$$\begin{array}{r} 21 \\ 21 \\ \hline 231 \end{array}$$

$\binom{22}{2} = \frac{22!}{2! \cdot 20!} = \frac{\overset{11}{\cancel{22}} \cdot 21 \cdot \cancel{20 \cdot \cdots \cdot 1}}{\cancel{2} \times 1 \times \cancel{20 \times \cdots \times 1}} = 231$

$\binom{22}{1} = 22$

$\cancel{22}$

\# of 5-card poker hands?

$\binom{52}{5} = \frac{52!}{5! \cdot 47!}$

$$\begin{array}{r} 324 \\ \hline 324 \\ 324 \\ \hline 3564 \end{array} \qquad \begin{array}{r} \times 21 \\ \hline 21 \\ 21 \\ \hline 231 \end{array}$$

5-card poker hands that are all hearts?

$\cancel{\binom{39}{13}}$ $\binom{13}{5}$

Draw 5 cards. $\mathbb{P}$[get a pair]

$$= \frac{\text{\# favorable}}{\text{total \#}} = \boxed{\frac{\binom{52}{5} - \frac{52\cdot 48\cdot 44\cdot 40\cdot 36}{5!}}{\binom{52}{5}}}$$

\# 5-card hands that do not have a pair? $= 52\times 48\times 44\times 40\times 36$

divide by 5! to remove order.

$$\underbrace{\binom{52}{5}}_{\text{unordered}} - \underbrace{52\times 48\times 44\times 40\times 36}_{\text{ordered 5-card hands}}$$