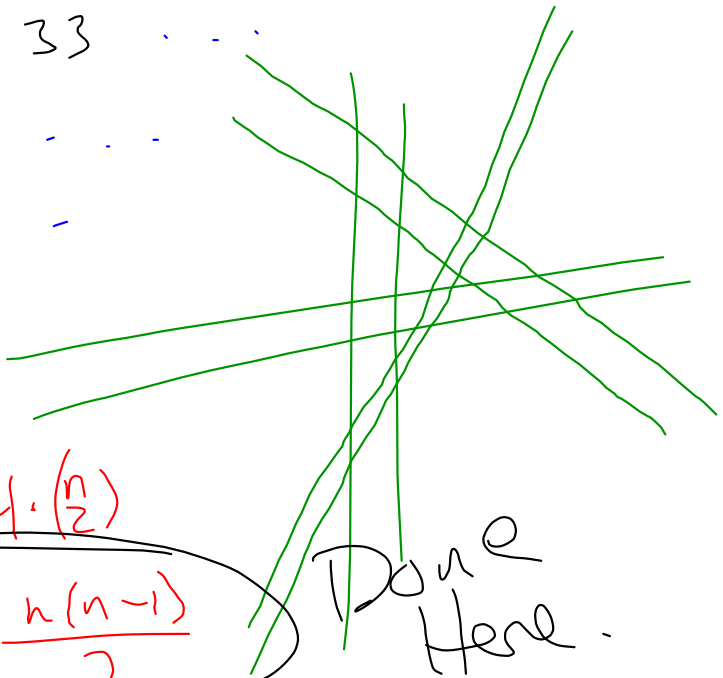
XC question:

Dividing the plane with n pairs of parallel lines

n	a_n
0	1
1	3
2	9
3	19
4	33

a_0	a_1	a_2	a_3	a_4	
1	3	9	19	33	...
	2	6	10	14	...
		4	4	4	...



$$a_n = 1\cdot\binom{n}{0} + 2\cdot\binom{n}{1} + 4\cdot\binom{n}{2}$$

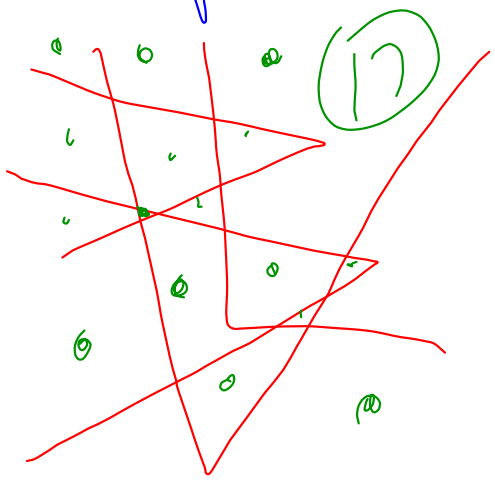
$$= 1 + 2n + 4\frac{n(n-1)}{2}$$

Done Here.

$$= 1 + 2n + 2n^2 - 2n = \boxed{2n^2+1}$$

Q: Into how many regions can n angles divide the plane?

Eg:

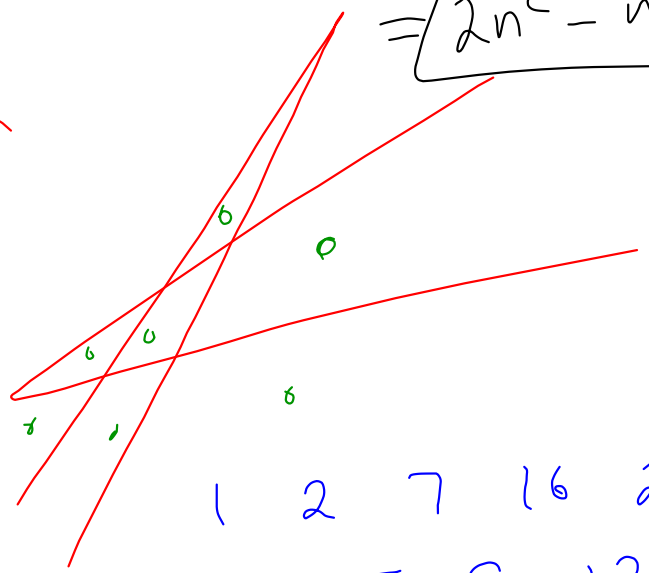


$$1\cdot\binom{n}{0}+1\cdot\binom{n}{1}+4\cdot\binom{n}{2}$$

$$= 1 + n + 4\,\frac{n(n-1)}{2}$$

$$= \boxed{2n^2 - n + 1}$$

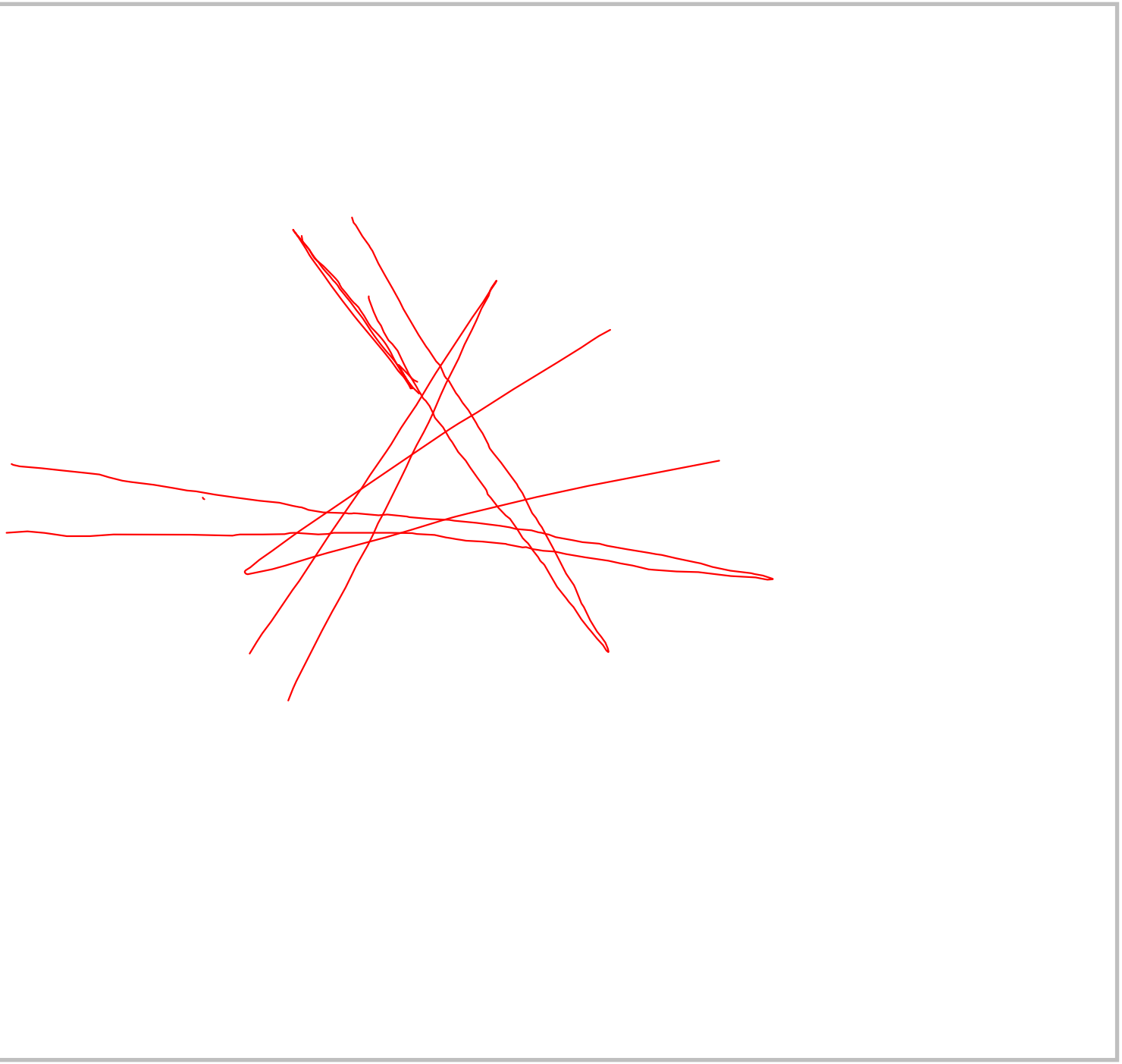
n	# reg
0	1
1	2
2	7
3	16
4	29
5	46
6	



1 2 7 16 29

1 5 9 13

4 4 4

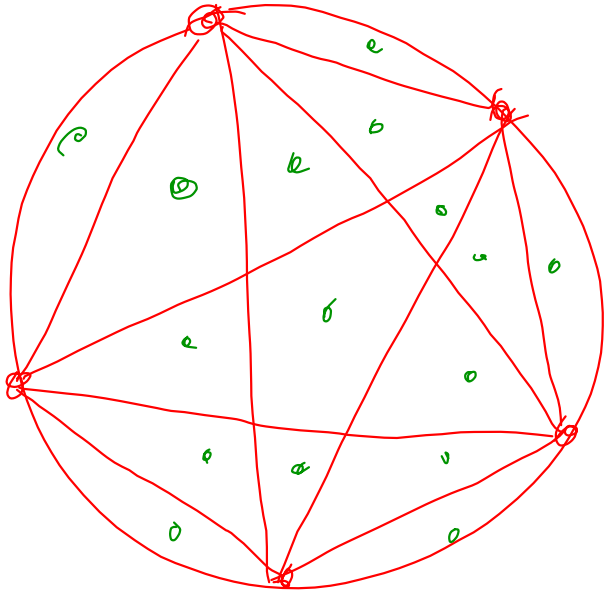


Title: Apr 23-10:23 AM (3 of 8)

XC +10 due @ final

Find formula for # regions you get by connect all pairs among n pts on a circle.

Eg: $n = 5$

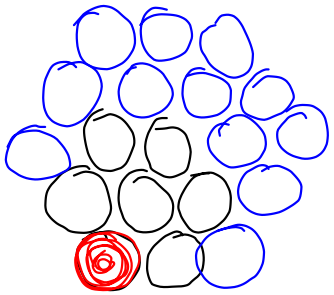


16 regions

(4^{th} degree polynomial.)

Pennies in a hexagon

Note this counter-intuitive number.



a_0	a_1	a_2	a_3	a_4
1	1	7	19	37
1	6	12	18	
0	6	6		
6				

$$a_n = 1 + 6\frac{n(n-1)}{2}$$

$$= 3n^2 - 3n + 1$$

Side	#
1	1
2	7
3	19
4	37

Q: How many ways to place 8 pennies in a row with no 2 consecutive Heads?

Solution: Do it for n pennies, $n \geq 1$

n	# ways
1	2
2	3
3	5
4	8
5	13
6	21
7	34
8	55

$n=1$: H, T

$n=2$: HT, TH, TT

$n=3$: TTT, HTT, THT, TTH, HTH

$n=4$:

0H: TTTT

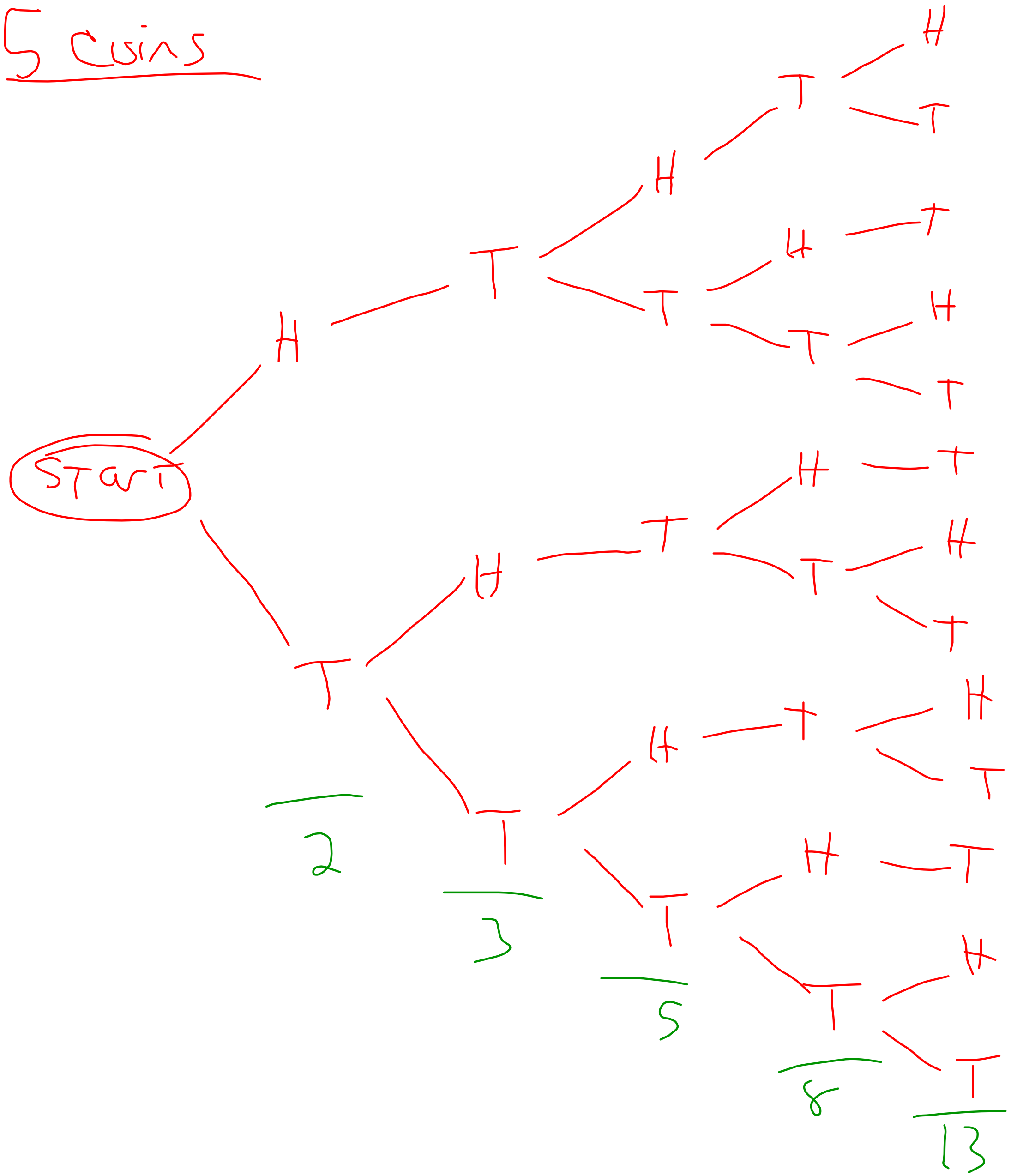
1H: HTTT, THTT, TTHT, TTTH

2H: HTHT, THTH, HTTH

3H: ∅

4H: ∅

5 coins



Each # is the sum of the 2 previous numbers

2 3 5 8 13 21 34 55

a_1 a_2 a_3 a_4 a_5

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55

1 2 3 5 8 13 21

$\Rightarrow$ No constant row

$\Rightarrow$ No polynomial formula

$$a_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{n+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+2}}{\sqrt{5}}$$

Binet's formula for the Fibonacci Numbers

Wed night 6-8 Review

HW: Prove this by induction.

Recurrence: $a_0 = 1$, $a_1 = 2$,

$a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$.