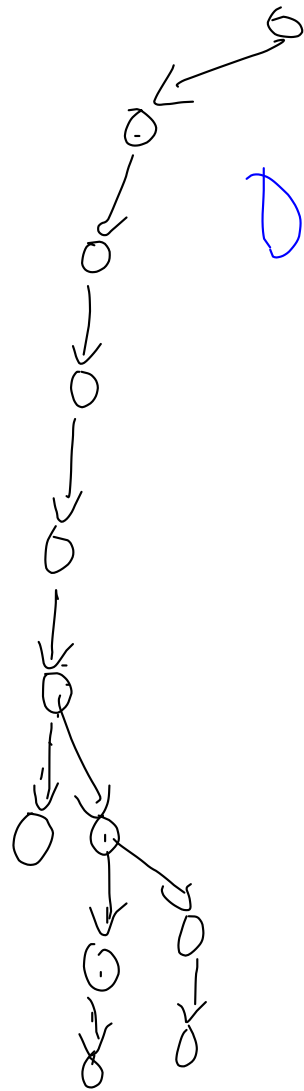
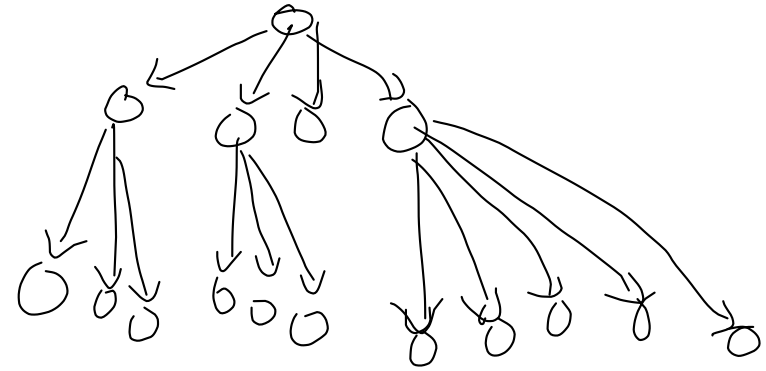


# Depth First Search



DFS Tree

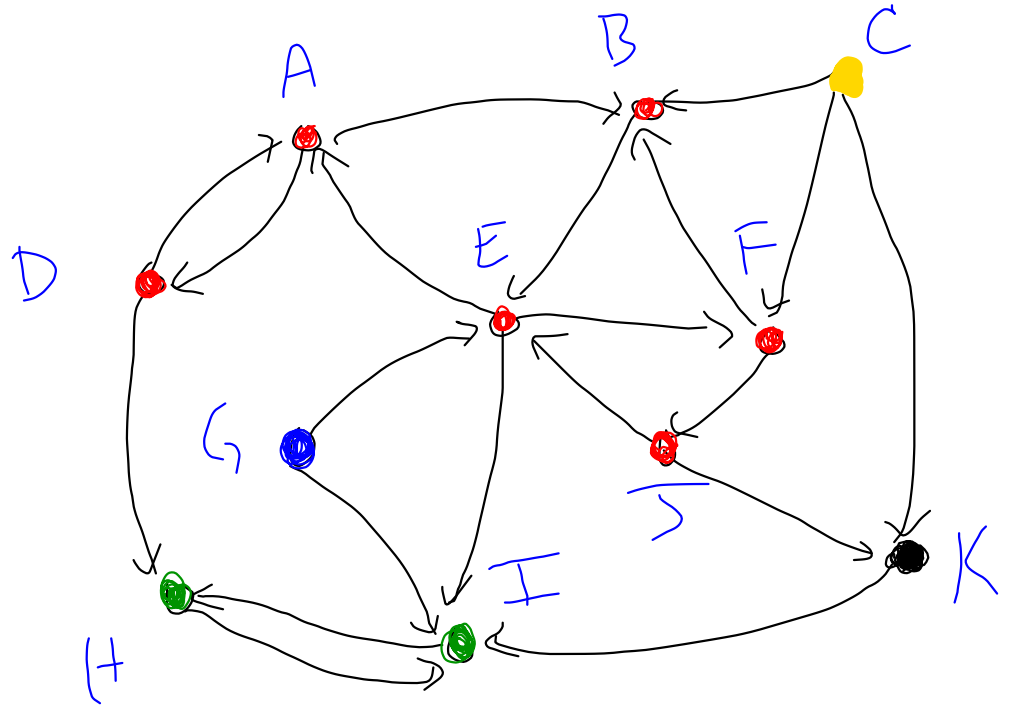
# Breadth-first search.



BFS Tree

Def: A strongly connected component in a digraph  $D$  is a maximal set  $C$  of vertices such that for each pair  $v, w$  of vertices in  $C$ , there is a directed path from  $v$  to  $w$ .

maximal = can't add any vertices to the set and stay strongly connected.



- If there is a cycle in  $D$ , then all vertices on that cycle are in the same SCC. (Strongly Connected Comp.)
- Any vertex with  $\text{indegree} = 0$  or  $\text{outdegree} = 0$  is a SCC unto itself.
- The relation "a ~ b" means "a and b are in a SCC together" is transitive

$$a \sim b \text{ and } b \sim c \longrightarrow a \sim c$$

- It's also symmetric and reflexive

$$a \sim b \longleftrightarrow b \sim a$$

$$a \sim a$$

Since " $\sim$ " is symmetric, reflexive and transitive, it breaks the vertices of  $D$  into Equivalence Classes = the SCCs.

# Algorithm for finding SCCs in a digraph

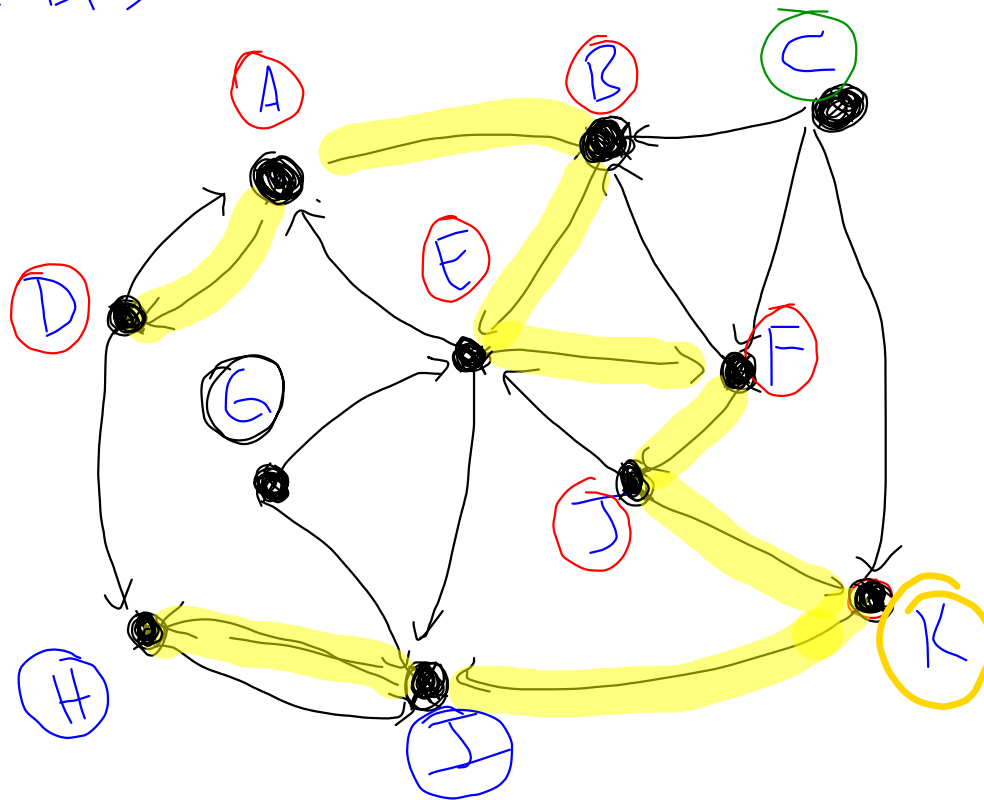
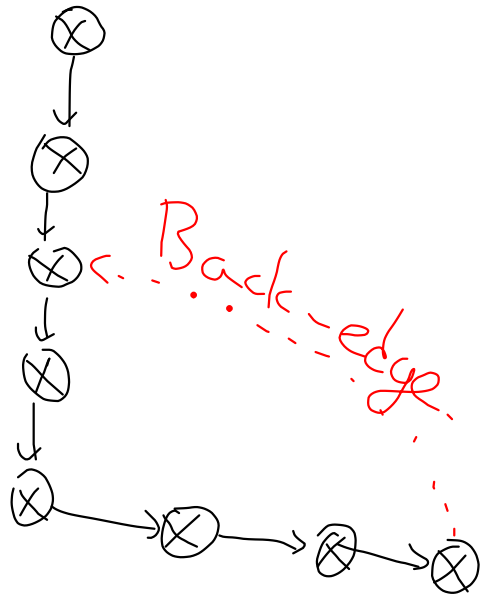
Lemma: The set of grey vertices

at any point in a DFS

form a directed

path from root

to current vertex.



Fact: A back edge gives a cycle,  
and all those vertices are in the  
same SCC.

# +10 XC

- Write up a precise description of how we just used DFS to find SCC.

- Write down the lemmas that need to be proved.

DFS

DFS.VISIT

added  
code

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# +10 XC

- prove those lemmas.

Due Wednesday. • 4/5/06