$$\boxed{\begin{array}{l} \text{for } i \leftarrow 1 \text{ TO } n-1 \\ \quad \text{for } j \leftarrow i \text{ TO } n \\ \quad\quad x{+}{+} \end{array}}$$

$i \qquad n$

$$\sum_{i=1}^{n-1} \Big[\underbrace{\sum_{j=i}^{n} 1}_{n-i+1} \Big]$$

$$\boxed{1+2+\cdots+k = \frac{k(k+1)}{2}}$$

$$= \sum_{i=1}^{n-1} (n-i+1)$$

Way 1

$$= \sum_{i=1}^{n-1} n - \sum_{i=1}^{n-1} i + \sum_{i=1}^{n-1} 1$$

$$= n \cdot (n-1) - \frac{(n-1)\cdot n}{2} + (n-1)$$

$$= n^2 - n - \frac{n^2}{2} + \frac{n}{2} + n - 1$$

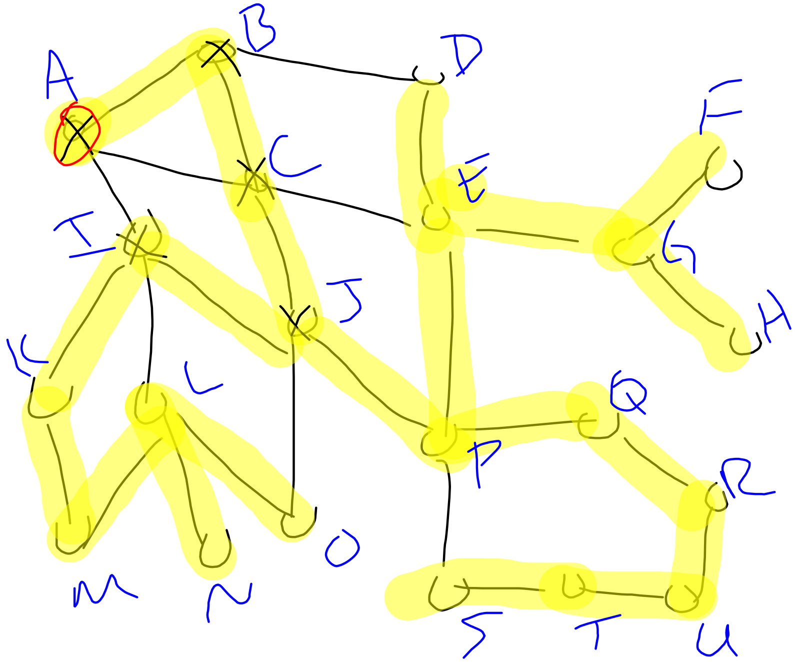
$$= \frac{n^2}{2} + \frac{n}{2} - 1 = \Theta(n^2)$$

Way 2

$$= n + (n-1) + (n-2) + \cdots + 3 + 2$$

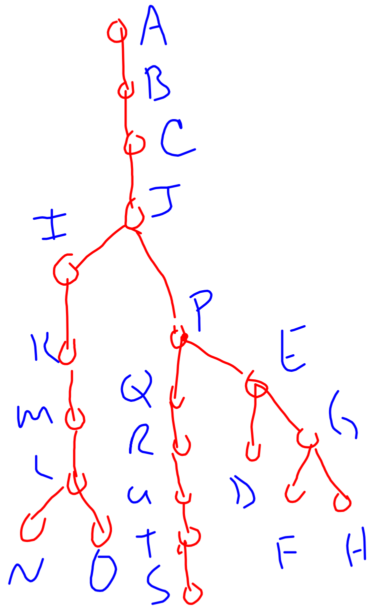
$= \text{Sum of \#s from } 2 \text{ TO } n$

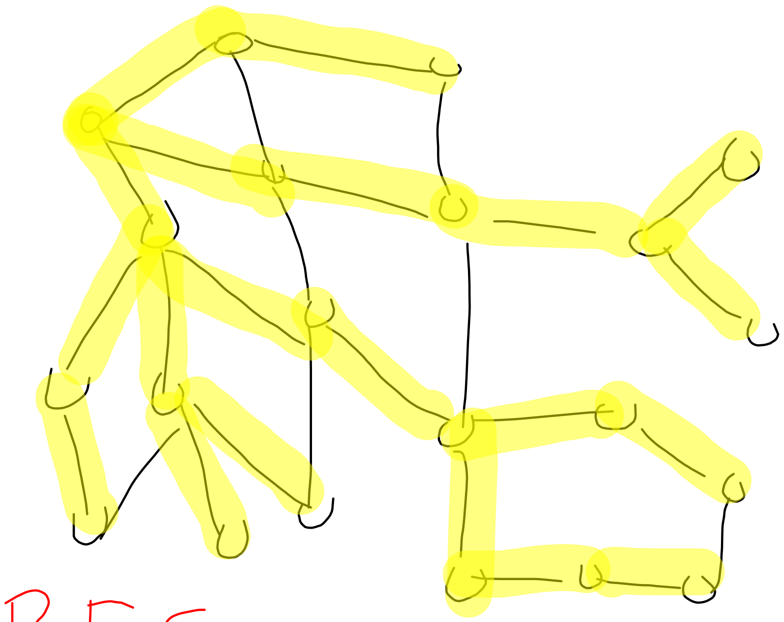
$$= \frac{n(n+1)}{2} - 1 = \Theta(n^2)$$



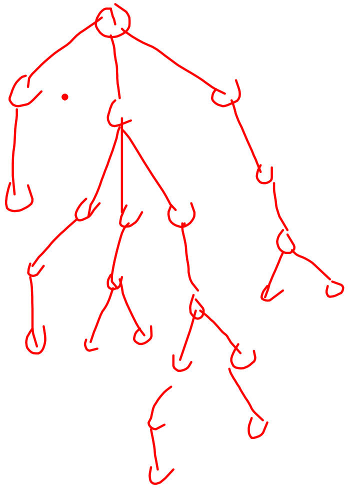
DFS

All black edges are back edges

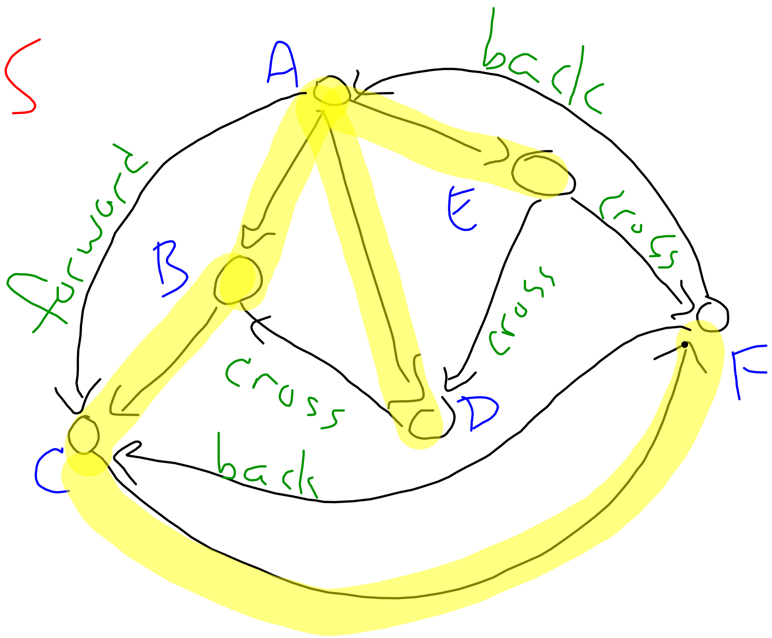




BFS



DFS



Alphabetical order – (how choose order of children)

A = ROOT.

nested discov/finish time intervals

forward edges: ancestor → descendant

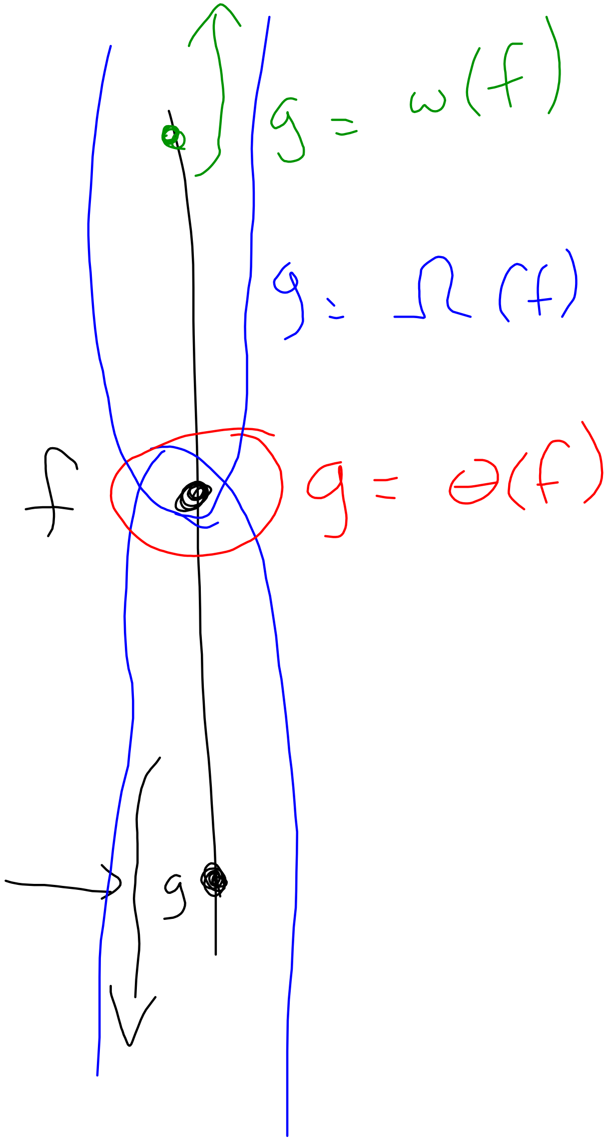
back edge: descendant → ancestor

$g = \omega(f) \to g = \Omega(f)$

$g = O(f)$

$g = o(f)$

$g = o(f) \Rightarrow g = O(f)$



Q 42 g: Prove that $n^2 + 6n + 5 = O(n^2)$

$$n^2 + 6n + 5 = n^2\left(1 + \frac{6}{n} + \frac{5}{n^2}\right)$$

$\frac{6}{n} \leq 1$ if $n \geq 6$

$\frac{5}{n^2} \leq 1$ if $n \geq 3$

So... $n^2 + 6n + 5 = n^2\left(1 + \frac{6}{n} + \frac{5}{n^2}\right) \leq n^2(1 + 1 + 1)$

$= 3n^2$.

for $n \geq 6 = n_0$

		x	y	y	x	y	y	x	x	y
	0	0	0	0	0	0	0	0	0	0
y	0	0	1	1	1	1	1	1	1	1
y	0	0	1	2	2	2	2	2	2	2
x	0	1	1	2	3	3	3	3	3	3
x	0	1	1	2	3	3	3	4	4	4
y	0									
y	0									
x	0									
x	0									

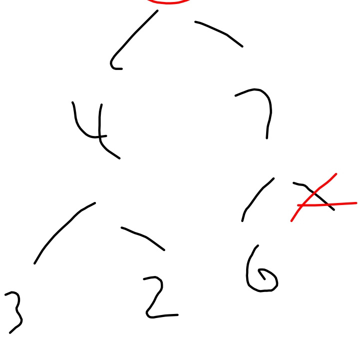
Extract(A)

A is a max heap.

= ~~8~~ 4 7 3 2 6 5

= 4 7 3 2 6 5

5 then max-heapify



heapsize - -

Knapsack problem

Q: What's the most value you can fit into a knapsack that holds max total weight W?

Items	weights	values
1	w_1	v_1
2	w_2	v_2
3	w_3	v_3
4	w_4	v_4
5	⋮	⋮
6		
7		
8		
⋮		
n	w_n	v_n

let $M(k, w)$ = most value you can get into a sack capable of weight w, from among items $1, 2, \ldots, k$.

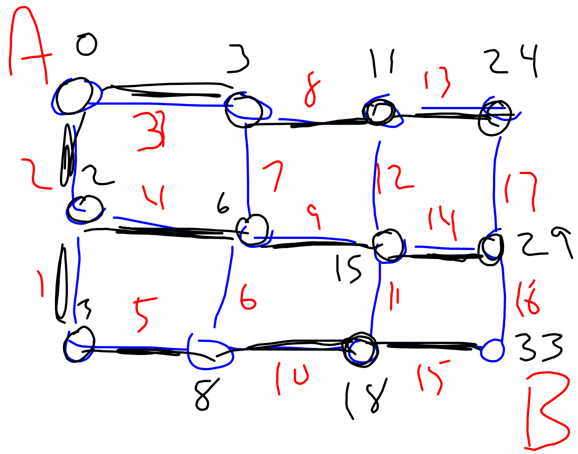
$$M(k,w) = \max \begin{cases} M(k-1, w) \\ M(k-1, w-w_k) + v_k \end{cases}$$

$$M(k,w) = \max \begin{cases} M(k-1, w) & \text{if we don't take item \#k} \\ M(k-1, w-w_k)+v_k & \text{if we do} \end{cases}$$

$\boxed{\log_2 16 = 4}$

Know: Shortest path on a dag.

$\log_{10} 1{,}000{,}000 = 6$



must move E or S.

$\log_2 \frac{1}{2} = -1$

$\log_3 81 = 4$

$$a^{\log_b c} = c^{\log_b a}$$

$$\boxed{T(n) = 5T\left(\frac{n}{2}\right) + n^2}$$

$$2^{\log_5 n} = \boxed{n^{\log_2 5}}$$

$\log_2 5 = 2.\ldots$

$$2^{\log_2 n} = n \qquad \boxed{2 < \log_2 5 < 3}$$