

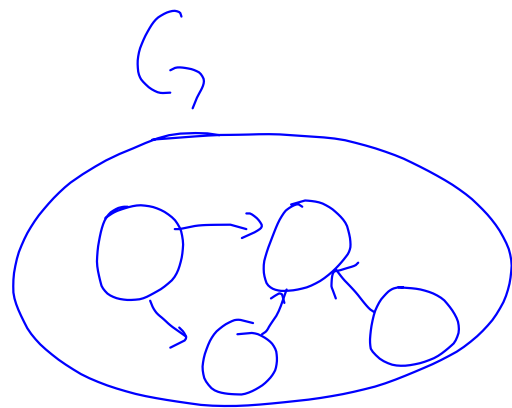
Good Morning!

Alg: (Find SCC)

- Do DFS on  $G$  to find finishing times  $f[v]$
- Create  $G^T$  (reversed edges)
- Do DFS on  $G^T$ , with the outermost loop of DFS run in order of finishing times, highest - to - lowest
- Each tree in this DFS is a SCC in  $G$ .

Some general Discussion.

Note what happens in  $G^T$  to SCC's.



A SCC in  $G$  is still a SCC in  $G^T$ , and  
vice-versa.

Def:  $u$  is a descendant of  $v$  in a DFS if in one of the DFS trees created by the DFS, there is a path from  $v$  to  $u$ .

Who are  $c$ 's descendants?

D, F, E, G, H

DFS forest

Tell me about the  $d + f$  times of vertices B and D.

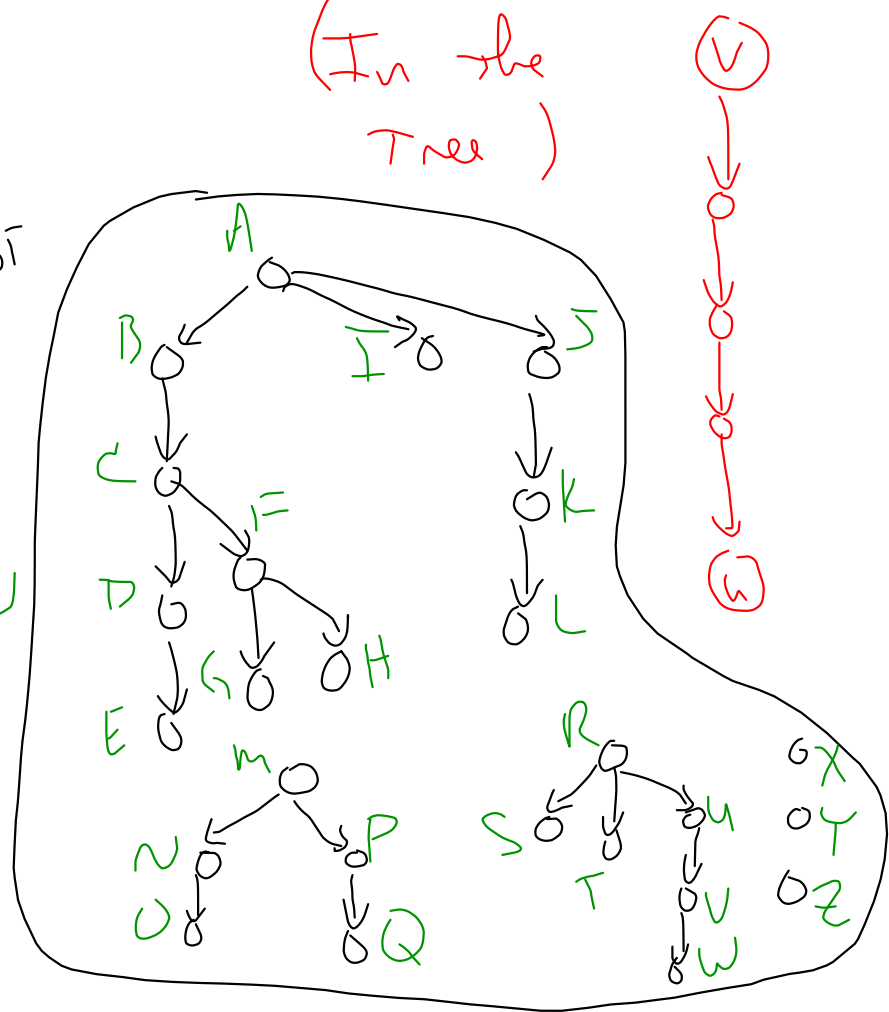
$$d[B] < d[D] < f[D] < f[B]$$

Thm: vertex  $u$  is a descendant of vertex  $v$  if and only if

$$d[v] < d[u] < f[u] < f[v]$$

(v (u u) v)

(In the Tree)



# White Path Theorem

If  $u$  is a descendent of  $v$ , then when  $v$  is first discovered, there is a path of white vertices from  $v$  to  $u$ , at that time. And vice-versa. (It's an if and only if thm).

forward direction is obvious.

how about vice-versa?

When  $v$  is discovered,  $u$  is white

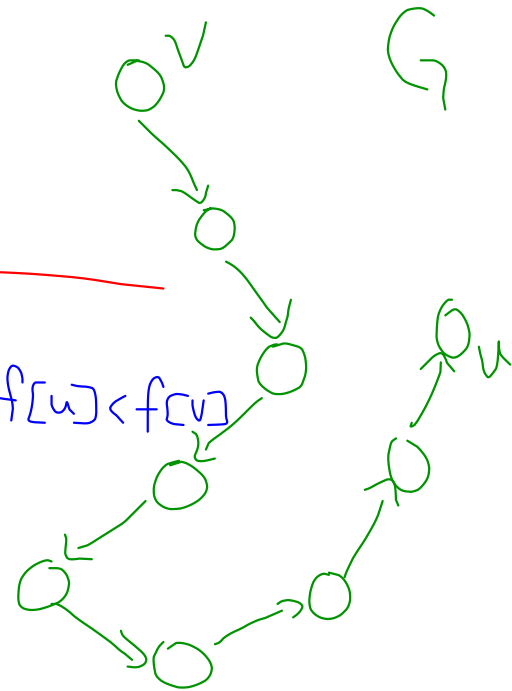
so  $d[v] < d[u]$

How about  $f[v] > f[u]$

$d[v] < d[u]$

$f[u] < f[v]$

Thus  
 $d[v] < d[u] < f[u] < f[v]$   
 $\rightarrow u$  is a descendent of  $v$ .



Def: If  $S$  is some set of vertices of a graph  $G$ ,

then  $d[S] = \text{earliest } d[v] \text{ for } v \in S$

$f[S] = \text{latest } f[v] \text{ for } v \in S$ .

Thm: If  $C$  and  $C'$  are SCCs in some

digraph  $G$ ,  $u \in C$ ,  $v \in C'$ , and  $(u, v)$  is an edge  
in  $G$ , then  $f[C] > f[C']$

Proof:

If  $d[C'] < d[C]$ , then we  
must finish all of  $C'$  before we

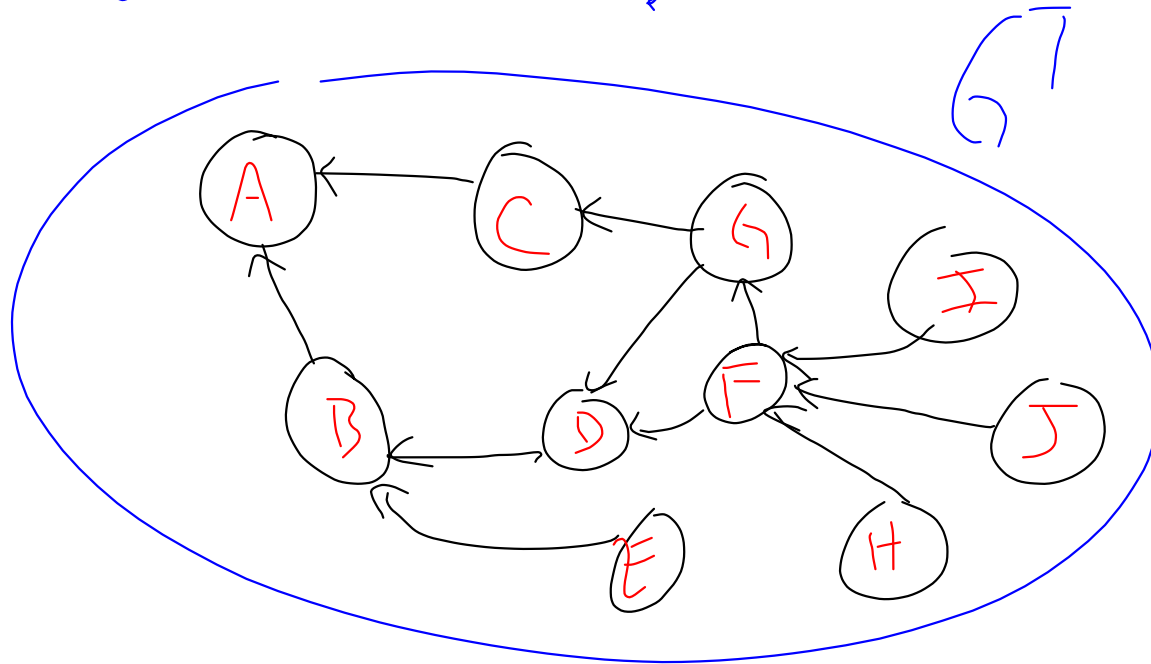
visit  $u$ , or else  $G^{SCC}$  has a cycle. Thus,  $f[C'] < f[C]$

But, if  $d[C] < d[C']$ , then all  $v' \in C'$  will be descendants  
of whichever vertex in  $C$  is first discovered, say  $u'$ .

thus  $f[v'] < f[u']$  for all  $v' \in C' \rightarrow f[C'] < f[C]$  ~~□~~



What does sorting the vertices by finishing times do on  $G^T$ ?



If  $(c) \rightarrow (c')$  in  $G^T$ , then  $f[c] < f[c']$