$T(n) = 2T(\sqrt{n}) + n.$

$T(2^k) = 2T(\sqrt{2^k}) + 2^k$

$T(2^k) = 2T(2^{\frac{k}{2}}) + 2^k$

(the "action" is in the exponent. let's talk about it)

Let $S(k) = T(2^k)$, substitute

(S deals with the exponent in T)

$S(k) = 2S(\frac{k}{2}) + 2^k$

Now, master theorem applies!

Compare $k^{\log_2 2}$ with 2^k

k^1 with 2^k

bigger!

$\sqrt{n} = n^{\frac{1}{2}}$

$T(n) = 2T(n^{\frac{1}{2}}) + n$

Let $n = 2^k$

(why "2" - no reason)

e^k, 5^k - ok.

$S(k) = \Theta(2^k)$

$T(2^k) = \Theta(2^k)$

$\boxed{T(n) = \Theta(n)}$

why not try this on a spreadsheet!

Eg: $T(3n+17) = 2T\left(3\cdot\frac{n}{7}+17\right) + n$

T is tripling and adding 17 to ~~the~~ $n, \frac{n}{7}$.

Let $S(n) = T(3n+17) \longleftrightarrow T(n) = S\left(\frac{n-17}{3}\right)$

Then $S(n) = 2S\left(\frac{n}{7}\right) + n$

Compare $n^{\log_7 2}$ with n^1

Winner!

$S(n) = \Theta(n)$

$T(n) = S\left(\frac{n-17}{3}\right) = \Theta\left(\frac{n-17}{3}\right) = \Theta(n)$.

$\cancel{T(n) = 2T(\sqrt{n}) + 1}$

$$T(n) = 2T(\sqrt[3]{n}) + 1$$

$$= 2T(n^{\frac{1}{3}}) + 1$$

$$T(2^k) = 2T(2^{\frac{k}{3}}) + 1$$

let $S(k) = \cancel{2}\, T(2^k)$

Recurrence becomes

$$S(k) = 2S\left(\frac{k}{3}\right) + 1$$

let $n = 2^k \longleftrightarrow k = \log_2 n$

By Master theorem

compare $k^{\log_3 2}$ with 1 $(= k^0)$

polynomially bigger

$$S(k) = \Theta(k^{\log_3 2})$$

$$T(2^k) = \Theta(k^{\log_3 2})$$

$$\boxed{T(n) = \Theta((\log_2 n)^{\log_3 2})}$$

$$T(n) \sim \Theta((\log n)^{0.63})$$

$T(n) = 2T(\sqrt{n}) + \log_2 n$

Start Same Way

$\boxed{\text{Let } n = 2^k, \Longleftrightarrow \log_2 n = k}$

Let $S(k) = T(2^k)$

$T(2^k) = 2T(2^{\frac{k}{2}}) + k$

$S(k) = 2S(\frac{k}{2}) + k$

this is the merge sort recurrence!

$S(k) = \Theta(k \log k)$

$T(2^k) = \Theta(k \log k)$

Back into "n"

$\rightarrow T(2^k)$

$T(n) = \Theta(\log_2 n \log \log n)$

$\boxed{T(n) = \Theta(\log n \cdot \log \log n)}$