The tree method: $T(n) = 2T\left(\frac{n}{2}\right) + n$ (Merge sort)

$(T(1) = 1)$

subproblems

size of subproblems

cost to combine sub-solutions.

$T(n)$

$T\left(\frac{n}{2}\right) \quad T\left(\frac{n}{2}\right) \quad + n$

n

$T\left(\frac{n}{2}\right) \quad T\left(\frac{n}{2}\right)$

$\rightarrow$

n

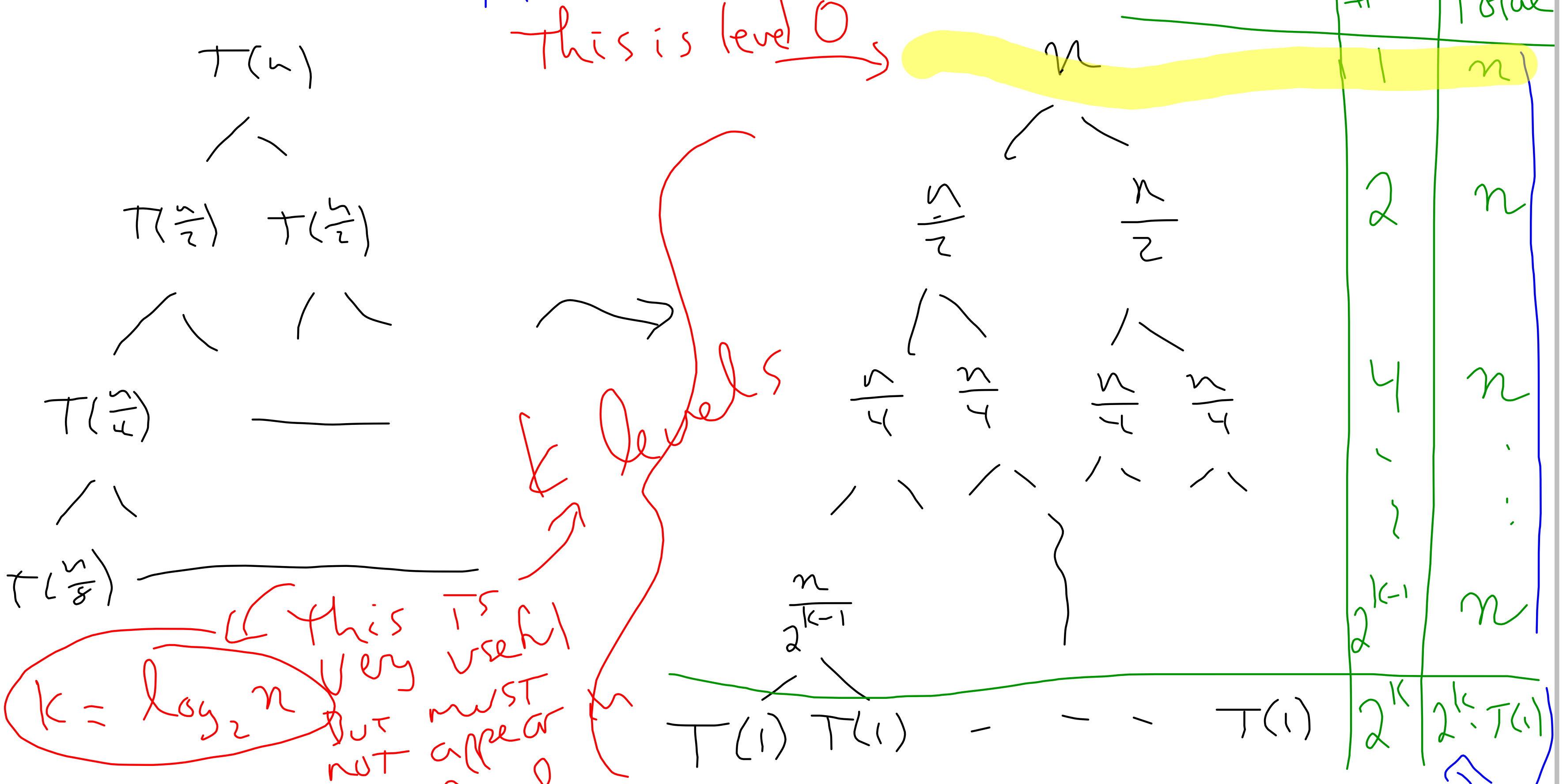
$\frac{n}{2} \quad \frac{n}{2}$

$T\left(\frac{n}{4}\right) \quad T\left(\frac{n}{4}\right) \quad \_\_\_\_$

— see next —

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

This is level 0



$k = \log_2 n$ ← this is very useful but must not appear in final answer.

Recall $T(1) = 1$

So, Total cost = sum of this column

$$= n \cdot (\cancel{k-1}\ k) + 2^k \cdot 1 = n \cdot \log_2 n + n$$

Let's analyze $n \log_2 n + n$

First of all – for Θ-concerns, the base of the log (as long as it's ≥ 1) is irrelevant!

$$\log_2 n = \frac{\log_b n}{\log_b 2}$$

for any base b.

$$= \frac{1}{\log_b 2}(\log_b n)$$

Constant

observe that $n = o(n \log_2 n)$

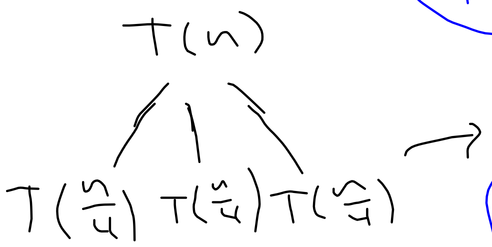
For HW, we showed

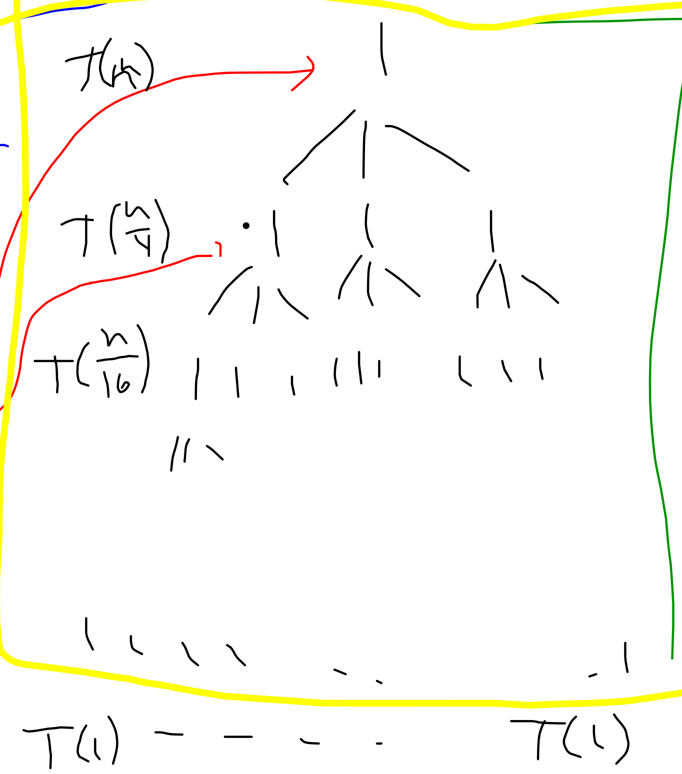
$$f + o(f) = \Theta(f)$$

So $n \log_2 n + n = \Theta(n \log n)$

Use tree method: $T(n) = 3T(\frac{n}{4}) + 1$

$T(1) = 5$





Cost of combining is always "1"

k levels

#	Row cost
1	1
3	3
9	9
27	27
3^{k-1}	3^{k-1}
3^k	$3^k \cdot T(1)$

$\boxed{k = \log_4 n}$

Total cost $= (1+3+9+\cdots+3^{k-1}) + 3^k \cdot 5$

Sum of geometric series $= \dfrac{\text{next} - \text{first}}{\text{ratio} - 1}$

$$= \frac{3^k - 1}{3-1} + 5 \cdot 3^k = \frac{1}{2}(3^k - 1) + 5 \cdot 3^k$$

— see next —

$$= \frac{3^k - 1}{3-1} + 5 \cdot 3^k = \frac{1}{2}(3^k - 1) + 5 \cdot 3^k$$

$(k = \log_4 n)$

$$= \frac{11}{2} \cdot 3^k - \frac{1}{2} = \Theta(3^k)$$

Must get in terms of "n"

$$= \Theta(3^k) = \Theta\left(3^{\log_4 n}\right) = \Theta\left(3^{\frac{\log_3 n}{\log_3 4}}\right)$$

$$= \Theta\left(n^{\frac{1}{\log_3 4}}\right) = \Theta\left(n^{\log_4 3}\right) \approx \Theta\left(n^{0.79}\right)$$

The Master Method

For recursions of form $T(n) = aT(\frac{n}{b}) + f(n)$

Compare	$f(n)$	with	$n^{\log_b a}$
$\Theta(f(n))$	$\boxed{f(n)}$	$>$	$n^{\log_b a}$
$\Theta(\boxed{f(n) \log n})$	$f(n)$	$=$	$n^{\log_b a}$
$\Theta(n^{\log_b a})$	$f(n)$	$<$	$\boxed{n^{\log_b a}}$

defined on friday