$T(n) = 3T(\lfloor \frac{n}{2} \rfloor) + 2$. Show $T(n) = \Omega(n)$

Assume $T(k) \geq c \cdot k$ for all $k < n$.

Consider $T(n)$

$T(n) = 3T(\lfloor \frac{n}{2} \rfloor) + 2 \geq 3 \cdot c \cdot \lfloor \frac{n}{2} \rfloor + 2 \geq 3c \cdot \frac{n-1}{2} + 2$

Say: we are subtracting a constant, but we have $\frac{3}{2}cn$, so it's $\geq c \cdot n$.

$= \frac{3c}{2} \cdot n + \left(2 - \frac{3c}{2}\right)$

Want $T(n) \geq c \cdot n$

$= c \cdot n \cdot \left(\frac{3}{2}\right) + \left(2 - \frac{3c}{2}\right)$

c could be 10000050!!!!!

$= c \cdot n \cdot 1 + \left(c \cdot n \cdot \frac{1}{2} + 2 - \frac{3c}{2}\right)$

Want $\geq c \cdot n$, eventually $\rightarrow \infty$

$\rightarrow T(n) = \Omega(n)$

Where is $T(n) = 3T(\frac{n}{2}) + 2$?

(Drop the floor)

Where does our recurrence lie?

$$n \quad < \quad n\log n \quad < \quad n\sqrt{n} \quad < \quad \frac{n^2}{\log n} \quad < \quad n^2$$

$n\sqrt{n} = n^{3/2}$

$\log(n^2) = 2\log n$

Let's use the Tree method!

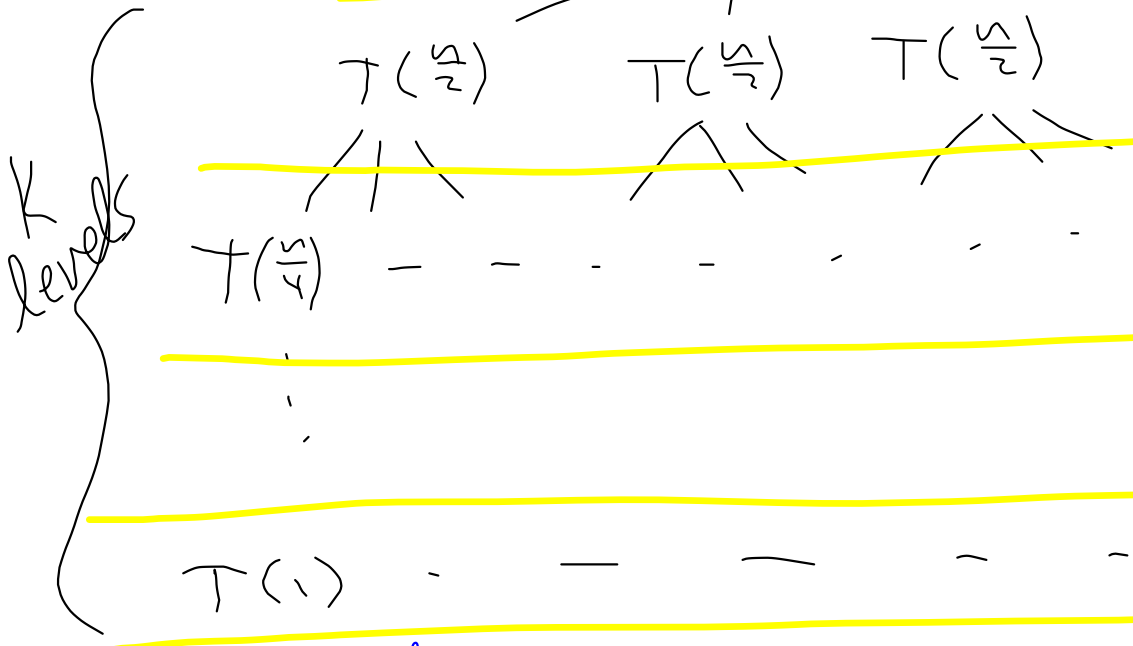
each is o(the next)

each is ω(the previous)

The Tree Method. ($T(n) = 3T(\frac{n}{2}), T(1) = 1$)

(Use a tree diagram to organize the "back substitution")

Let's try a simpler one first: $T(n) = 3T(\frac{n}{2})$



T(what)	# copies	Total
$T(n)$	1	$T(n)$
$T(\frac{n}{2})$	3	$3T(\frac{n}{2})$
$T(\frac{n}{4})$	9	$9T(\frac{n}{4})$
⋮	⋮	⋮
$T(1)$	3^k	$3^k \cdot 1$

Total Time

$T(1)\ 3^k$ $3^k \cdot 1$

$k = \log_2 n$

Need answer in terms of n.

$$T(n) = 3^{\log_2 n} \cdot 1 =$$

$$= 3^{\frac{\log_3 n}{\log_3 2}} = \left(3^{\log_3 n}\right)^{\frac{1}{\log_3 2}}$$

$$= n^{\frac{1}{\log_3 2}} \approx n^{1.58}$$

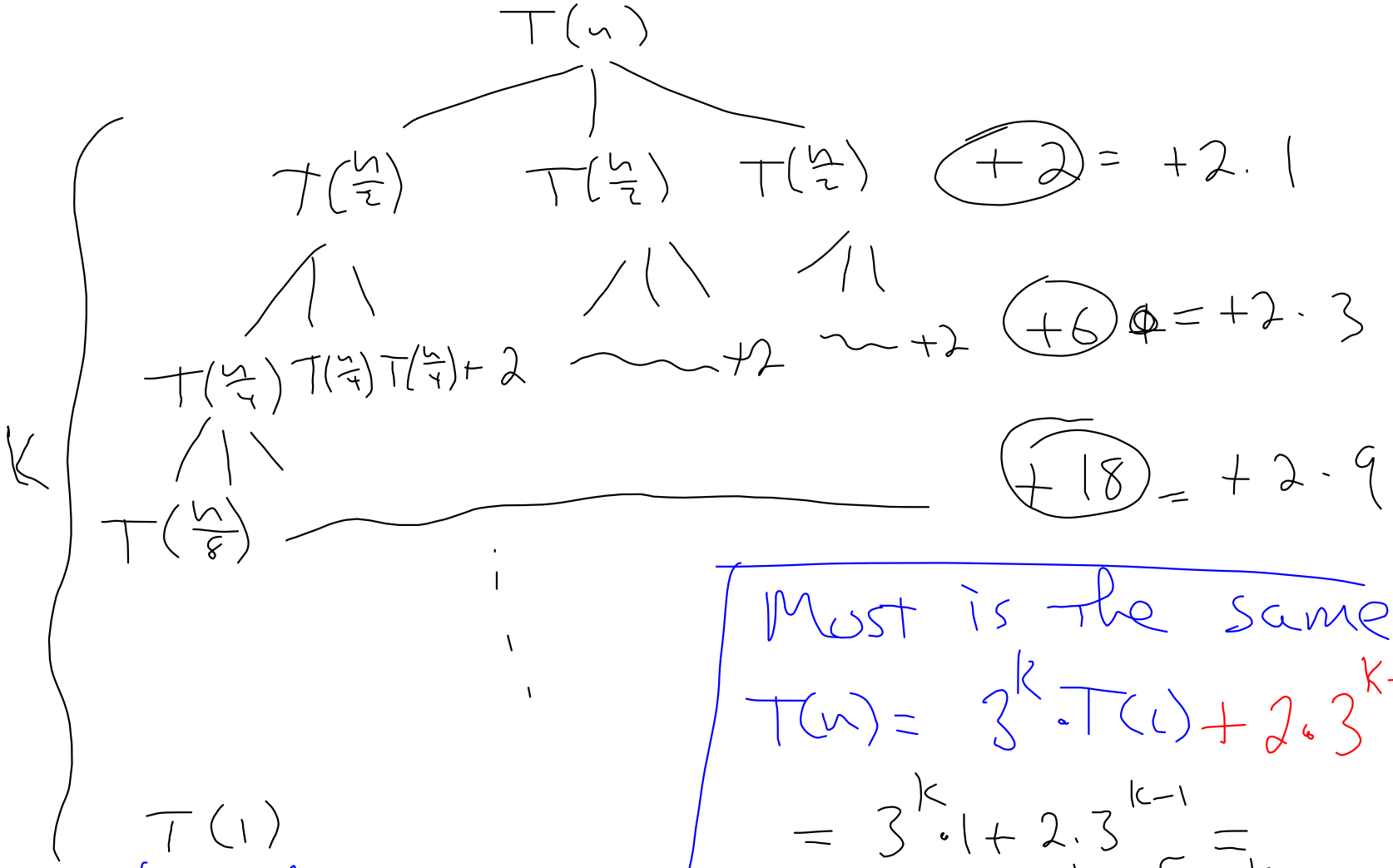
Change of base

$$\log_b n = \frac{\log_k n}{\log_k b}$$

Want $\log_3 n$ up there

$$\log_3 2 = \frac{\ln 2}{\ln 3}$$

What about $T(n) = 3T(\frac{n}{2}) + 2$?



Must is the same

$T(n) = 3^k \cdot T(1) + 2 \cdot 3^{k-1}$

$= 3^k \cdot 1 + 2 \cdot 3^{k-1} =$

$= 3^k + \frac{2}{3} \cdot 3^k = \frac{5}{3} \cdot 3^k$

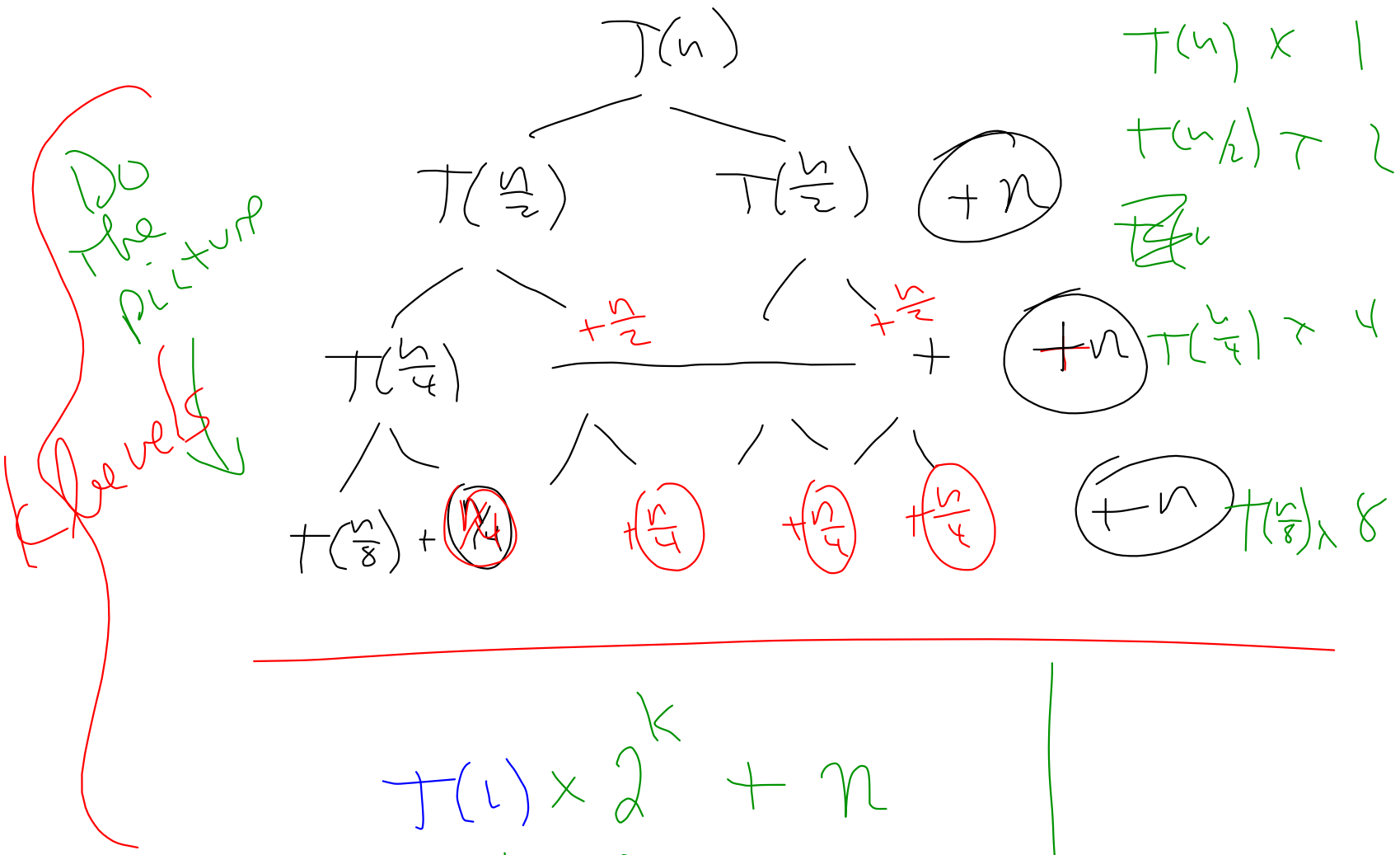
$$= 3^k \cdot 1 + 2 \cdot 3^{k-1} =$$
$$= 3^k + \frac{2}{3} \cdot 3^k = \frac{5}{3} \cdot 3^k$$

$$= \frac{5}{3} \times \text{Previous answer, on slide 4.}$$

$$= \frac{5}{3} * n^{\log_2 3} \approx \frac{5}{3} \cdot n^{1.58} = \Theta(n^{1.58\ldots})$$

multiplication

$T(n) = 2T(\frac{n}{2}) + n$ (Merge Sort)



$T(1) \times 2^k + n$

1×2

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