Discrete Mathematics **Name**_____
Quiz 1a **February 9, 2005**

Assuming that $A \rightarrow B$, $\neg C \rightarrow D$ and $C \rightarrow A$ are all true, show that $\neg D \rightarrow B$ is true.

Negate the following expression: $(A \vee B) \rightarrow C$

Build a truth table for the expression in the previous problem

Discrete Mathematics **Name**_____
Quiz 1b **February 9, 2005**

Assuming that $A \vee B$, $A \rightarrow C$ and $\neg B$ are all true, show that C is true.

Negate the following expression: $C \rightarrow (A \vee B)$

Build a truth table for the expression in the previous problem

Discrete Mathematics **Name**_____
Quiz 1c **February 9, 2005**

Assuming that $A \vee B$, $\neg B \vee C$ and $\neg C \vee D$ are all true, show that $A \vee D$ is true.

Negate the following expression: $(A \rightarrow B) \vee C$

Build a truth table for the expression in the previous problem

Discrete Mathematics **Name**_____
Quiz 1d **February 9, 2005**

Assuming that $A \wedge B$, $A \rightarrow C$ and $B \rightarrow D$ are all true, show that $C \wedge D$ is true.

Negate the following expression: $C \vee (A \rightarrow B)$

Build a truth table for the expression in the previous problem

Discrete Mathematics **Name**_____
Quiz 2a **February 16, 2005**

1. How many subsets does the set {1, 2, 3, 4, 5} have?

2. How many anagrams does "NOODLE" have?

3. If A = {1, 2, 3} and B = {2, 3, 4, 5}, what is:
   a. $A \cap B$

   b. $A \cup B$

   c. $A \setminus B$

Discrete Mathematics **Name**_____
Quiz 2b **February 16, 2005**

1. How many subsets does the set {a, b, c, d, e, f} have?

2. How many anagrams does "CHEESE" have?

3. If A = {1, 2, 3, 4, 5} and B = {2, 3, 4}, what is:
   a. $A \cap B$

   b. $A \cup B$

   c. $A \setminus B$

Discrete Mathematics **Name**_____
Quiz 2c **February 16, 2005**

1. How many subsets does the set {v, w, x} have?

2. How many anagrams does "MEATBALL" have?

3. If A = {b, c, d, e} and B = {a, b, c}, what is:
   a. $A \cap B$

   b. $A \cup B$

   c. $A \setminus B$

Discrete Mathematics **Name**_____
Quiz 2d **February 16, 2005**

1. How many subsets does the set {2, 4, 6, 8} have?

2. How many anagrams does "LINGUINI" have?

3. If A = {r, s, t} and B = {p, q, r, s, t, u, v}, what is:
   a. $A \cap B$

   b. $A \cup B$

   c. $A \setminus B$

Discrete Mathematics **Name**_____
Exam I **March 9, 2005**

This test has 18 questions worth a total of 120 points. Please make sure that you have them all. 100 points is considered a perfect score.

1. Which syllogism is being used in each of these cases? Demonstrate by labeling propositions and rewriting the argument with your labels. Also give explicitly the name of the syllogism.

   a. Lori: Sven is either clueless or lying. And I know he's not clueless.

      Scott: So he must be lying!

   b. Mary: Honest people are trusted, but people don't trust Cybil.

      Betsy: So Cybil is not honest.

   c. Komo: If you speak loudly, people will hear you. And if people hear you, then people will see your point of view.

      Niko: So if you speak loudly, people will see your point of view.

   d: Mr. Gotti: Throw the game, Tommy, or we'll give you cement shoes

      Officer Olaf: You better not throw the game or you'll get kicked out of baseball.

      Tommy: So, I either get cement shoes or I get kicked out of baseball.

4. Given that $\neg A$, $\neg X$, $P \vee A$ and $\neg P \vee B$ are all true, show that $\neg(B \rightarrow X)$ is true.

5. Use truth tables to show that $\neg(p \vee \neg q)$and $\neg p \wedge q$ are logically equivalent.

6. Negate each of the following expressions:
   a. $p \rightarrow \neg q$

   b. $(p \wedge q) \vee (p \leftrightarrow q)$

   c. $\forall t\, P(t)$

7. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 3, 4\}$. True or False (if you say "False", say why):
   a. $A \subseteq B$

   b. $B \subseteq A$

   c. $\{3, 4\} \subseteq A$

   d. $\{3, 4\} \subseteq B$

   e. $\{3, 4\} \in A$

   f. $\{3, 4\} \in B$

8. Compute each of the expressions below, where $A = \{\varnothing, 1, 2, 3\}$ and $B = \{2, 3, 4, 5\}$.
   a. $A \cup B$

   b. $A \cap B$

   c. $A \setminus B$

9. Compute the power set of each of the following sets.
   a. $\varnothing$

   b. $\{1, 2\} \cup \{2, 3\}$

   c. $\{a, b\}$

10. What is the cardinality of the power set of a set with n elements?

11. Use membership tables to prove that for all sets A and B, $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (B \cap A)$

12. What is $|\{x \in \mathbb{N} \mid -5 \leq x \leq 7\}|$? Your answer should be an integer.

13. How many ways are there to put 10 quarters into 3 pockets if:
    a. Pockets may be empty?

    b. Each pocket must get at least one quarter?

    c. The first pocket must get at least one quarter and the second pocket must get at least 2 quarters?

14. Draw rows 0 to 8 of Pascal's Triangle and in your triangle, circle entries 7 Choose 2 and 6 Choose 6.

15. Prove by mathematical induction that $\forall n \geq 1, 6 + 12 + 18 + 24 + \cdots + 6n = 3n(n + 1)$

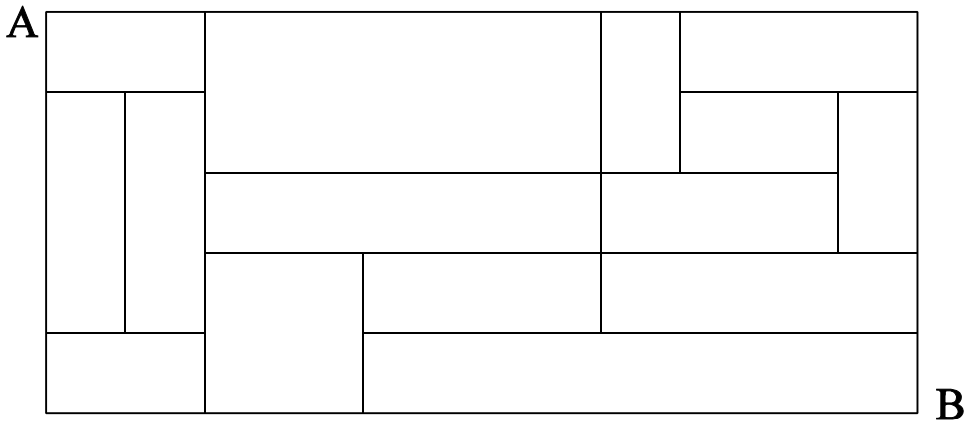
16. Some expansion questions:
    a. What is the coefficient of a^5b^2 in the expansion of $(2a + b)^7$?

    b. What is the coefficient of a^3b^6 in the expansion of $(2a + b)^8$?

    c. What is the coefficient of a^3b^6c in the expansion of $(2a + b - c)^{10}$?

17. How many ways are there to walk from A to B in the figure below without backtracking?

18. The "Who's the Boss" sequence: 1, 2, 1, 3, 1, 2, ...



    a. Give the first 30 terms

    b. How many terms appear before the first "9" appears?

    c. The "Bosses Difference" sequence is obtained by taking the difference between consecutive terms in the "Who's the Boss" sequence. It starts 1, –1, 2, –2, 1, –1, 3, –3, 1, –1, ...
        i. What is the sum of the first 1000 terms of the "Bosses Difference" sequence?

        ii. How many terms appear before the first "8" appears in the "Bosses Difference" sequence?

Discrete Mathematics **Name**_____
Quiz 4a **March 30, 2005**

1. A standard six-sided die is tossed.
   a. What is the probability that a “1" comes up?

   b. What is the probability that a “1" doesn’t come up?

   c. What is the outcome set?

2. A standard six-sided die is tossed 5 times:
   a. What is the probability that the first toss is even?

   b. What is the probability that the first toss is not even?

   c. What is the probability that an even number never appears among the five tosses of the die?

   d. What is the probability that an even number does appear among the five tosses of the die?

3. Two balls are drawn at random from an urn containing 3 red balls and 5 blue balls. What is the probability that the have the same color?

Discrete Mathematics **Name**_____
Quiz 4b **March 30, 2005**

1. A standard six-sided die is tossed.
   a. What is the probability that a “2" comes up?

   b. What is the probability that a “2" doesn’t come up?

   c. What is the outcome set?

2. A standard six-sided die is tossed 4 times:
   a. What is the probability that the first toss is odd?

   b. What is the probability that the first toss is not odd?

   c. What is the probability that an odd number never appears among the five tosses of the die?

   d. What is the probability that an odd number does appear among the five tosses of the die?

3. Two balls are drawn at random from an urn containing 4 red balls and 5 blue balls. What is the probability that the have the same color?

Discrete Mathematics **Name**_____
Quiz 4c **March 30, 2005**

1. A standard coin is tossed.
   a. What is the probability that Heads comes up?

   b. What is the probability that Heads doesn't come up?

   c. What is the outcome set?

2. A standard six-sided die is tossed 6 times:
   a. What is the probability that the first toss is odd?

   b. What is the probability that the first toss is not odd?

   c. What is the probability that an odd number never appears among the six tosses of the die?

   d. What is the probability that an odd number does appear among the six tosses of the die?

3. Two balls are drawn at random from an urn containing 3 red balls and 4 blue balls. What is the probability that the have the same color?

Discrete Mathematics **Name**_____
Quiz 4d **March 30, 2005**

1. A standard coin is tossed.
   a. What is the probability that Tails comes up?

   b. What is the probability that Tails doesn't come up?

   c. What is the outcome set?

2. A standard six-sided die is tossed 6 times:
   a. What is the probability that the first toss is even?

   b. What is the probability that the first toss is not even?

   c. What is the probability that an even number never appears among the five tosses of the die?

   d. What is the probability that an even number does appear among the five tosses of the die?

3. Two balls are drawn at random from an urn containing 3 red balls and 6 blue balls. What is the probability that the have the same color?

Discrete Mathematics **Name**_____
Exam II **April 13, 2005**

This test has 12 questions. Please make sure that you have them all.

1. A coin is tossed 10 times.
   a. What is the probability that the first toss is Heads?

   b. What is the probability that the 5^{th} toss is Heads?

   c. What is the probability that the last toss is Heads?

   d. What is the probability that Heads comes up exactly 3 times?

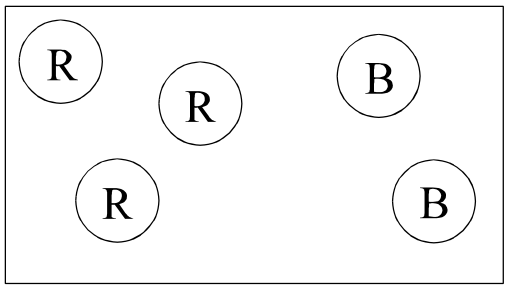
   e. What is the probability that no two consecutive tosses are the same?

   f. What is the probability that exactly "HTTHTHTTTH" comes up?

   g. What is the probability that the first and last toss are the same?

   h. What is the probability that Heads appears more than Tails?

2. An urn contains 3 red balls and 2 blue balls.



a. If a ball is drawn at random from the urn, what is the probability that it is red?

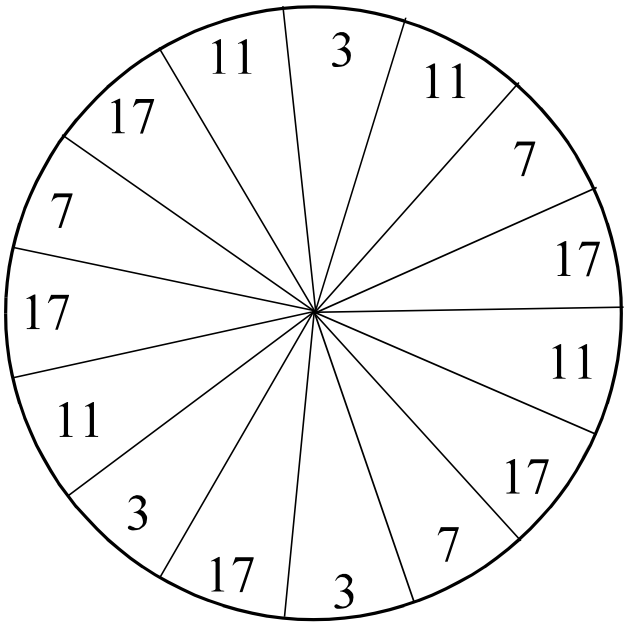
b. Two balls are drawn from the urn, without replacement. What is the probability that they are both red?

c. Two balls are drawn from the urn, without replacement. What is the probability that they are the same color?

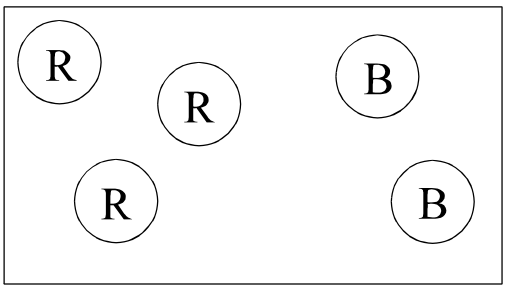
d. The emperor decides the fate of a prisoner by drawing a ball randomly from that urn, noting the color and then replacing it, and repeating this for a total of 7 draws. The prisoner lives if any of the draws yield a blue ball. What is the probability that the prisoner lives?

e. Three balls are taken out of the urn and placed on the table. After this is done, what is the probability that both the urn and the table contain a blue ball?

3. In the "**Prime Time**" carnival game a player spins the spinner to the right and wins the amount that comes up on his spin. What is the expected winnings for this game?



4. A ball is randomly selected from the same urn, the color noted, and then replaced, and this is repeated for a total of 3 draws. What is the expected number of red balls that will be drawn?



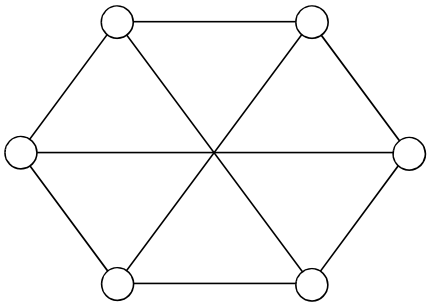
5. Alice and Bob each select a random integer from 1 to 30. What is the probability that the sum of the digits of Alice's number is the same as the sum of the digits of Bob's number?

6. What is the probability that a randomly tossed die comes up "1"?

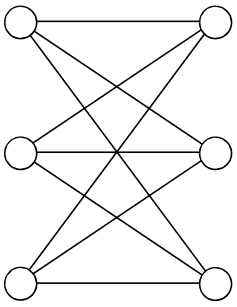
7. A pair of dice is tossed. What is the probability of getting doubles? That is, what is the probability that the two dice show the same number?

8. What is the probability that a 5-card poker hand does not contain a pair?

9. Two of the graphs below are isomorphic to each other, and the other one is not isomorphic to either of them.



A



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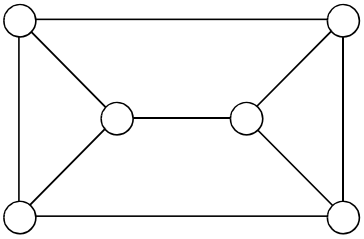


C

a. Which pair is isomorphic? Prove your answer by labeling the vertices in an appropriate way. (Note that there are extra copies of this graphic on the last page.)

b. Prove that the remaining one is not isomorphic to one of the others by finding a structural difference between them.

10. How many different degree sequences are possible on a graph with 3 vertices? List all the degree sequences and show a graph for each one. (Include connected and disconnected graphs.)

11. For each of the degree sequences shown below, give a graph that has that degree sequence (no multigraphs, please) or give a reason that the given sequence cannot be the degree sequence of a graph.

a. 2, 2, 2

b. 3, 2, 1

c. 3, 3, 2, 2

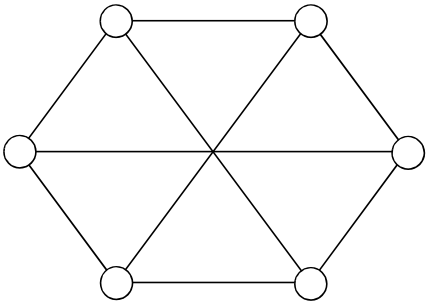
d. 7, 1, 1, 1, 1, 1, 1, 1

e. 6, 3, 2, 2, 2, 2, 1, 1, 1, 1

f. 5, 5, 4, 3, 2, 2, 2, 2, 2, 1

12. A certain molecule consists entirely of carbon and oxygen atoms.
    a. There are 20 carbon atoms.
    b. Each carbon atom is connected to 4 other atoms, any combination of carbon and/or oxygen
    c. Each oxygen atom is connected to 2 other atoms, any combination of carbon and/or oxygen
    d. 4 of the carbons are connected to only carbon atoms
    e. 12 of the carbons are connected to 3 carbon atoms and 1 oxygen atom
    f. 4 of the carbons are connected to 2 carbon atoms and 2 oxygen atoms
    g. 8 of the oxygen atoms are connected to 2 other oxygen atoms
    h. 4 of the oxygen atoms are connected to 1 oxygen atom and 1 carbon atom
    i. The rest of the oxygen atoms are connected only to carbon atoms.

    How many oxygen atoms are there?



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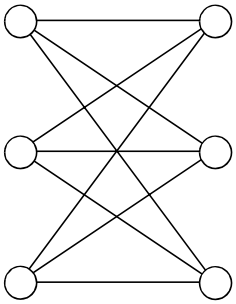

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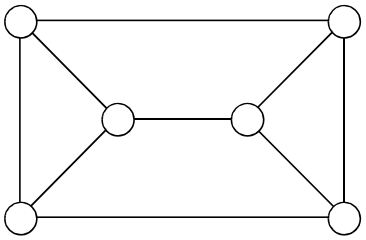

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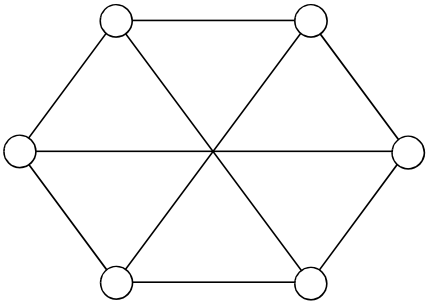

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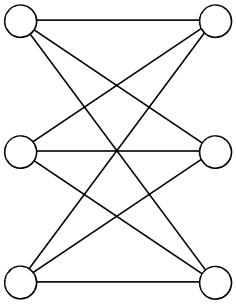

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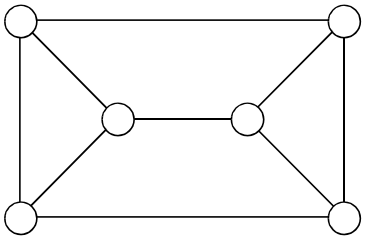

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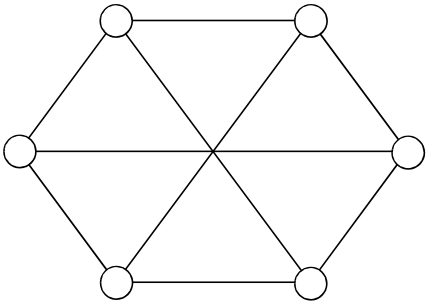

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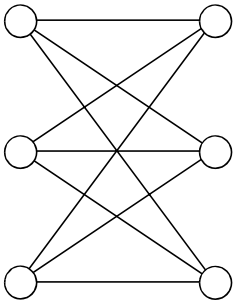

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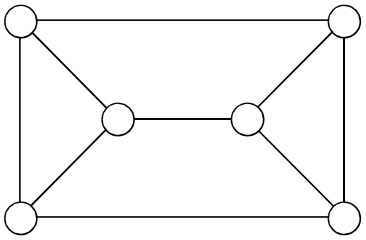

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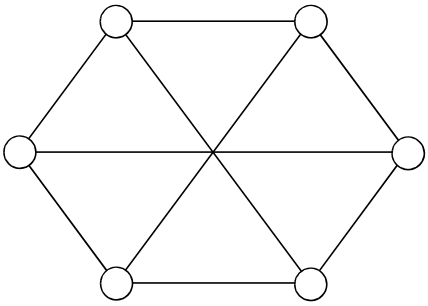

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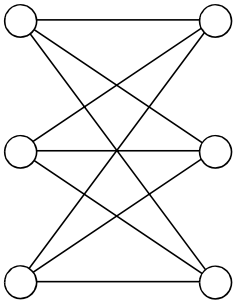

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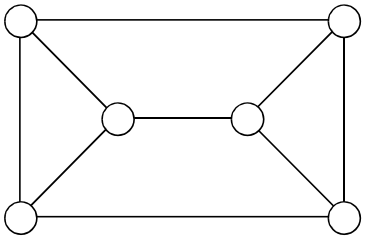

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