

Introduction to Probability

What is the probability that a fair coin comes up heads when flipped?

What is the probability of getting an even number when you roll a die?

What is the probability that a randomly-dealt poker hand (5 cards) contains no aces?

A man rolls a pair of dice 24 times, and wins if he ever gets boxcars (double sixes; sum = 12).

What is the probability that he wins?

What is the probability that there are two people in this room with the same birthday?

Experiments, Outcome Sets and Events

Experiment: An activity which produces some result, called an outcome

Outcome Set: The set of possible outcomes of an experiment. (Each trial must produce exactly one outcome from this set.)

Event: A subset of the outcome set

For example:

Experiment: Roll a die

Outcome Set: $\{1, 2, 3, 4, 5, 6\}$

Events:

| | |
|-------------|------------------|
| Get a 1 | $\{1\}$ |
| Less than 5 | $\{1, 2, 3, 4\}$ |
| Even number | $\{2, 4, 6\}$ |

Experiments, Outcome Sets and Events

The probability of an event is the sum of the probabilities of the outcomes that comprise it.

For example:

Experiment: Draw a card and look at the suit

Outcome Set: { , , , }

Events: Get a heart { }
Red card { , }
Don't get a spade { , , }

| Event | Probability |
|-------------------|-------------|
| Get a heart | |
| Red card | |
| Don't get a spade | |

Equally Likely Events

If the outcome set contains n elements and each outcome is equally likely, then each outcome has probability $1/n$.

An urn (there has to be an urn...) contains 20 balls (yes, they have to be balls...) numbered 1 to 20. What is the probability of drawing a multiple of 5?

$$\{ \textcircled{5}, \textcircled{10}, \textcircled{15}, \textcircled{20} \}$$

Toss a coin 4 times. What is the probability that heads appeared an even number of times?

$$\{0 \text{ heads}, 2 \text{ heads}, 4 \text{ heads}\}$$

(Bad Example!)

Be Clear about the Outcome Set!

When a coin is tossed 4 times, what are you looking for? Number of heads or the H-T sequence? Either is a valid choice, but you need to be clear on this when performing the experiment.

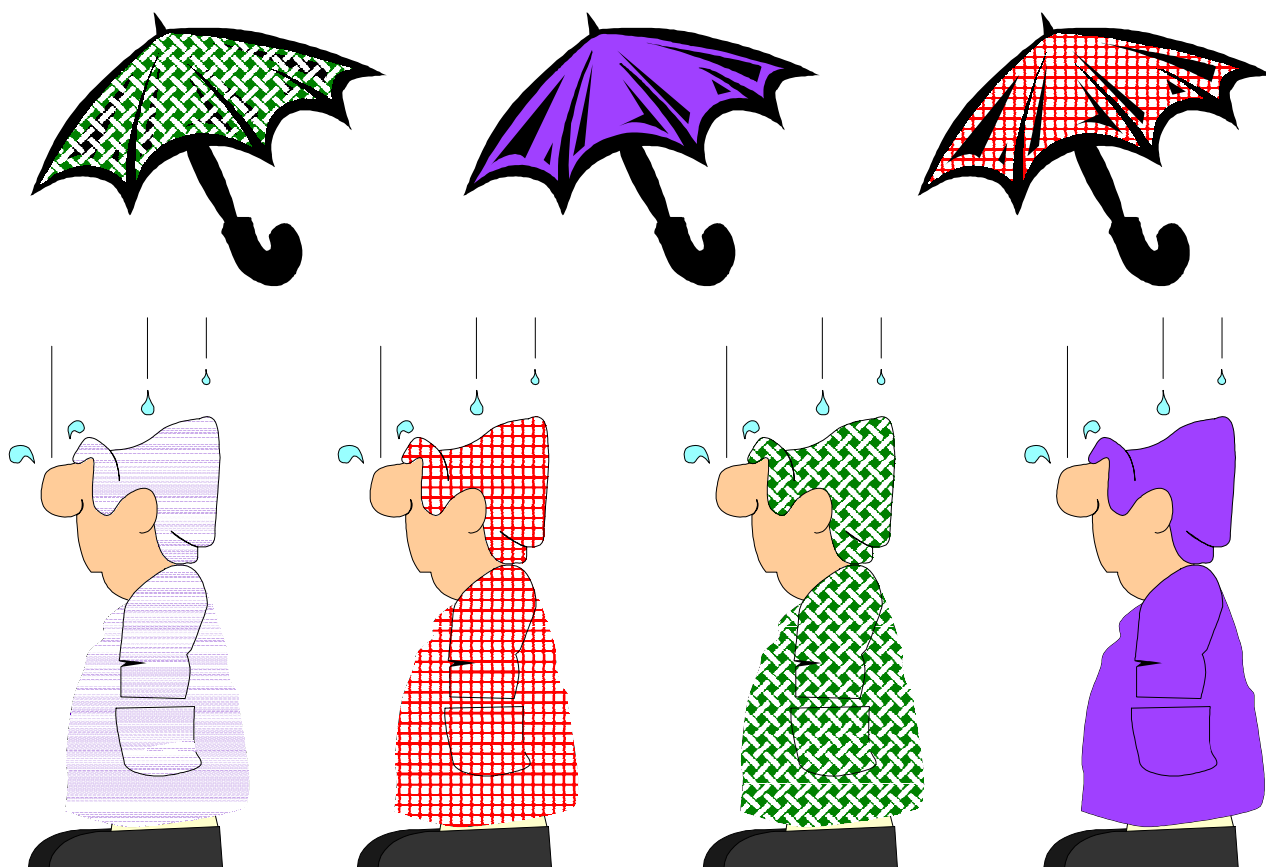
| Experiment | Outcome Set |
|-----------------|--|
| Number of Heads | {0, 1, 2, 3, 4} |
| H-T Sequence | {HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT} |

In the first case, the outcomes are not equally likely. In the second case, they are.

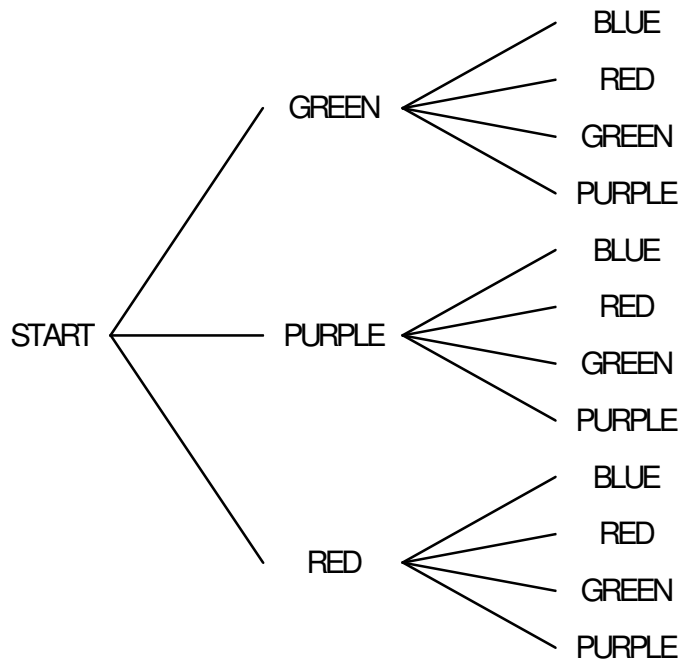
| 0 | 1 | 2 | 3 | 4 |
|------|------------------------------|--|------------------------------|------|
| TTTT | HTTT THTT TTHT TTTH | HHTT HTHT HTTH THHT THTH TTHH | HHHT HHTH HTHH THHH | HHHH |

Umbrellas and Raincoats

Stuart Mojado has 4 raincoats and 3 umbrellas to choose from as he heads out into the rain. What is the probability that he selects the same color umbrella as raincoat, if he selects each randomly?



Umbrellas and Raincoats



When the outcomes are equally likely, the probability of an event can be computed by:

$$\frac{\text{Size of Outcome set}}{\text{Size of Event}}$$

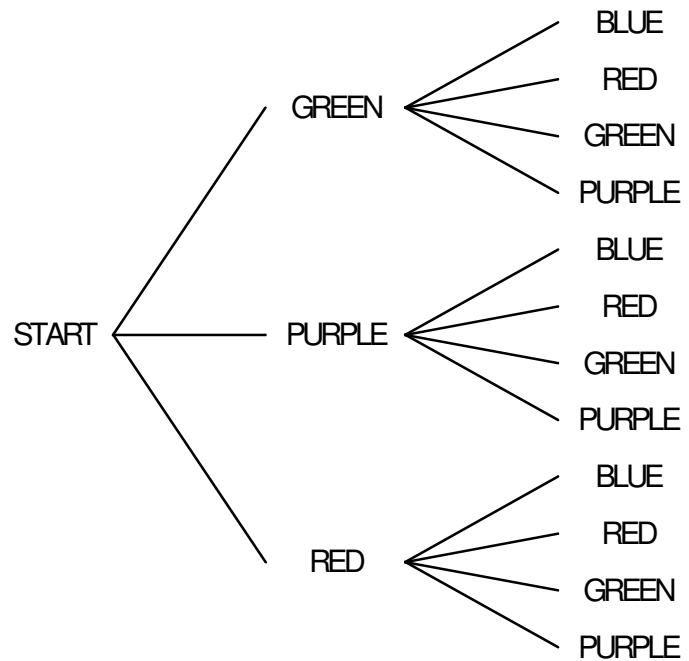
Which is more commonly written:

$$\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The Multiplication Rule for Probability

What is the probability that Stuart selects a red raincoat, but not a red umbrella?

We can use the tree diagram and the previous rule to find the probability



Or we can use the multiplication rule:

If an **event** consists of several independent parts
And each part must occur to complete the **event**
Then the probability of the **event** can be figured
by multiplying together the probabilities of each
part

The Multiplication Rule for Probability

What is the probability that Stuart selects a red raincoat, but not a red umbrella?

Event:

"Selects red raincoat **and** selects non-red umbrella"

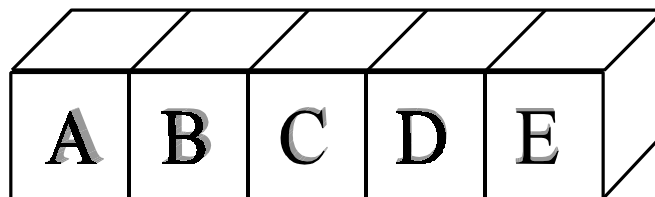
Use the Multiplication Rule:

$$\text{Prob(Selects red raincoat)} = 1/4$$

$$\text{Prob(Selects non-red umbrella)} = 2/3$$

$$\text{Prob(Event)} = 1/4 \times 2/3 = 2/12 = 1/6$$

Five blocks, labeled A, B, C, D and E, are lined up randomly on a bookshelf. What is the probability that the "A" comes first and the "E" comes last?



Probability Practice

One card is drawn at random from a standard deck of cards. What is the probability that...

- a. ...it is a heart?
- b. ...it is not a club?
- c. ...it is a 3?

A pair of dice is tossed.

What is the probability that...

- d. ...the first die comes up "6?"
- e. ...the second die is different than the first?
- f. ...the sum is even?

A coin is tossed five times.

What is the probability that...

- g. ...the third toss is heads?
- h. ...the first and last tosses are both tails?
- i. ...the tosses all come up the same?
- j. ...heads comes up exactly three times?

Virginia Pick 3 Lottery

| PLAY TYPE | EXAMPLE | ODDS | PRIZE |
|--|-------------------------|------------|-------|
| EXACT ORDER PLAY: | 123 | 1 in 1,000 | \$500 |
| ANY ORDER PLAYS: | | | |
| 6 - WAY | 123,132,213,231,312,321 | 1 in 166.7 | \$80 |
| 3 - WAY | 112,121,211 | 1 in 333.3 | \$160 |
| 50/50 PLAYS: | | | |
| 50/50 6-WAY | | | |
| ANY ORDER PORTION | 132,213,231,312,321 | 1 in 200 | \$40 |
| EXACT ORDER PORTION | 123 | 1 in 1,000 | \$290 |
| <i>NOTE: Exact order portion includes payout from any order portion.</i> | | | |
| 50/50 3-WAY | | | |
| ANY ORDER PORTION | 122,221 | 1 in 200 | \$80 |
| EXACT ORDER PORTION | 112 | 1 in 1,000 | \$330 |
| <i>NOTE: Exact order portion includes payout from any order portion.</i> | | | |
| COMBO PLAYS: | | | |
| 6-WAY (costs \$6) | 123,132,213,231,312,321 | 1 in 166.7 | \$500 |
| 3-WAY (costs \$3) | 112,121,211 | 1 in 333 | \$500 |
| PAIRS PLAYS: | | | |
| FRONT PAIR | 12X | 1 in 100 | \$50 |
| SPLIT PAIR | 1X3 | 1 in 100 | \$50 |
| BACK PAIR | X23 | 1 in 100 | \$50 |

Disclaimer

Discrete Teaching
does not necessarily
approve of the
implementation of
state lotteries

Virginia Mega Millions

Select 5 numbers out of 52, and then select another ball out of 52, which may be one of the original 52.

| MATCH | | MATCH | PRIZE | CHANCES |
|---|---|-------|-----------|------------------|
| 5 | + | 1 | Jackpot | 1 in 135,145,920 |
| 5 | + | 0 | \$175,000 | 1 in 2,649,920 |
| 4 | + | 1 | \$5,000 | 1 in 575,089 |
| 4 | + | 0 | \$150 | 1 in 11,276 |
| 3 | + | 1 | \$150 | 1 in 12,502 |
| 2 | + | 1 | \$10 | 1 in 833 |
| 3 | + | 0 | \$7 | 1 in 245 |
| 1 | + | 1 | \$3 | 1 in 152 |
| 0 | + | 1 | \$2 | 1 in 88 |
| Overall chances of winning any prize: 1 in 43 | | | | |

Virginia Cash 5

Select 5 numbers out of 34.

| MATCH | WAGER | PRIZE | CHANCES |
|--------|--------|-----------|--------------|
| 5 of 5 | \$1.00 | \$100,000 | 1 in 278,256 |
| | \$.50 | \$50,000 | |
| | \$.25 | \$25,000 | |
| 4 of 5 | \$1.00 | \$100 | 1 in 1,919 |
| | \$.50 | \$50 | |
| | \$.25 | \$25 | |
| 3 of 5 | \$1.00 | \$5.00 | 1 in 69 |
| | \$.50 | \$2.50 | |
| | \$.25 | \$1.25 | |

Arizona "The Pick" Lottery

| Match | Prize | Odds |
|--------------------------------|----------|----------------|
| 6 of 6 | Jackpot | 1 in 4,496,388 |
| 5 of 6 + Bonus Ball | \$10,000 | 1 in 749,398 |
| 5 of 6 | \$1,000 | 1 in 22,041 |
| 4 of 6 | \$40 | 1 in 503.8 |
| 3 of 6 | \$2 | 1 in 34.35 |

Pick 6 numbers out of 41

"With one less ball in the mix - 41 instead of 42 - you now have a better chance at winning the jackpot!"

The Choose Numbers and Probability

Having the anagram formula and the choose numbers available allows us to ask much more interesting probability questions!

A single die is rolled 12 times. What is the probability that each number comes up twice?

What is the probability that a coin tossed 5 times will yield exactly two heads?

What is the probability that a coin tossed 10 times yields exactly 7 heads?

Deck of Cards

What is the probability that a randomly dealt five-card poker hand will contain a pair (two cards of the same rank)?

What is the probability of getting one pair, but not any 3-of-a-kind nor two pair?

| | | | |
|----|----|----|----|
| 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 |
| 7 | 7 | 7 | 7 |
| 8 | 8 | 8 | 8 |
| 9 | 9 | 9 | 9 |
| 10 | 10 | 10 | 10 |
| J | J | J | J |
| Q | Q | Q | Q |
| K | K | K | K |
| A | A | A | A |