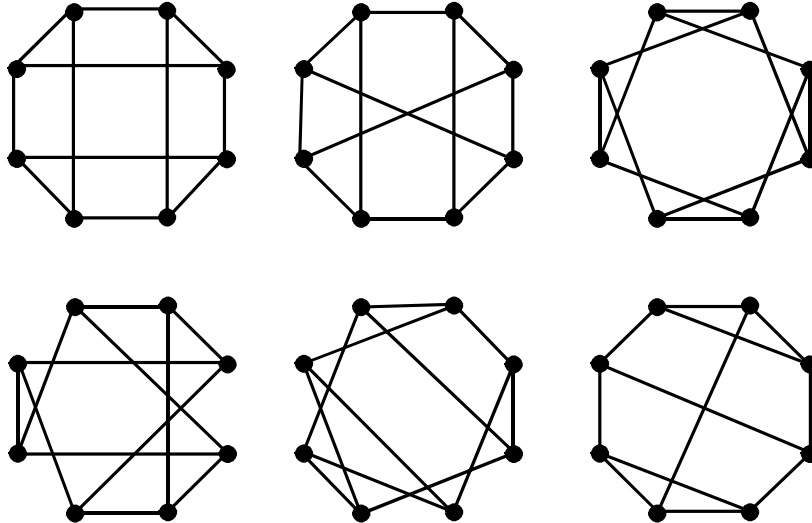
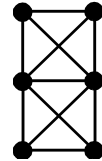


Graph Theory Exercises

1. The six graphs below consist of three pairs of isomorphic graphs. Which are the isomorphic pairs?



2. Is it possible to draw the graph shown to the right without crossings, using only straight lines?

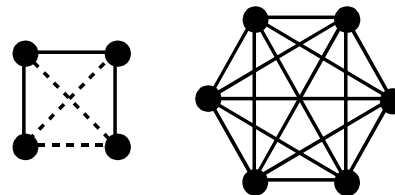


3. If you flip a coin three times, there are 8 ways the coins can come up: HHH, HHT, HTH, etc... Suppose these are the labels on 8 vertices of a graph, and that you draw an edge between a pair of vertices if the labels differ in exactly one place. For example, HTH will be adjacent to HTT, but will not be adjacent to THH. To which of the graphs in problem 1 will the resulting graph be isomorphic?
4. The *degree sequence* of a graph is the sequence of the degrees of the vertices of that graph sorted from greatest to smallest. For example, the degree sequence of the graph in problem 2 is 5, 5, 3, 3, 3, 3. For each of the following, decide whether or not it can be the degree sequence of a graph. If you say it can be, draw an appropriate graph. If you say it cannot be, give a reason why.
- 3, 3, 3, 3
 - 4, 4, 4, 4
 - 3, 3, 3, 2, 1
 - 1, 1, 1, 1, 1, 1
 - 5, 4, 4, 3, 2, 2
 - 4, 3, 3, 2, 2, 1
 - 4, 4, 3, 3, 3, 3, 3, 3, 2, 2

5. What are all possible degree sequences of a graph with 4 vertices? Include those cases where the graph might be disconnected, too.

6. Prove that it is impossible for the degree sequence of a graph (whether or not it's connected) to have no repeats.

7. The complete graph on 4 vertices (shown to the right) has been decomposed into 2 paths containing all the edges. This isn't the only way to do the job, but it is a particularly nice way, because the paths both have the same length.



a. Can you decompose the 15 edges of K_6 (shown to the right), into 3 paths, each of length 5?

b. Can you decompose the 28 edges of K_8 , the complete graph on 8 vertices, into 4 paths, each of length 7?

c. (**Bonus problem**) Can you prove that for all n , the edges of the complete graph K_n on n vertices can be covered by the paths all of the same length?

8. There are 7 guests at a formal dinner party, and the host wishes to have each person shake hands with each other person, for a total of 21 handshakes altogether. The host also wishes the following rules to be obeyed:

- The handshakes should take place sequentially.
- Each handshake (except the first) should involve someone from the previous handshake.
- No person should be involved in 3 consecutive handshakes.

Is there a way to sequence the handshakes so that these conditions are all met?
How about for 3, 4, 5, 6 or 8 people?

9. Sam the sculptor wishes to make a "World Peace" sculpture, and this is his idea: He will sculpt 7 pillars, standing for the 7 continents, and will place them in a circle. Then he will string gold thread between the pillars so that each pillar is connected to exactly 3 other pillars. (Sam explained the reason for the "3," but the explanation is not important here.) Can you see the reason that, although Sam might have good intentions, his sculpture is just physically impossible?

10. There are some men and 15 women in a room. Each man shook hands exactly 6 women, and each woman shook hands with exactly 8 men. How many men are in the room?