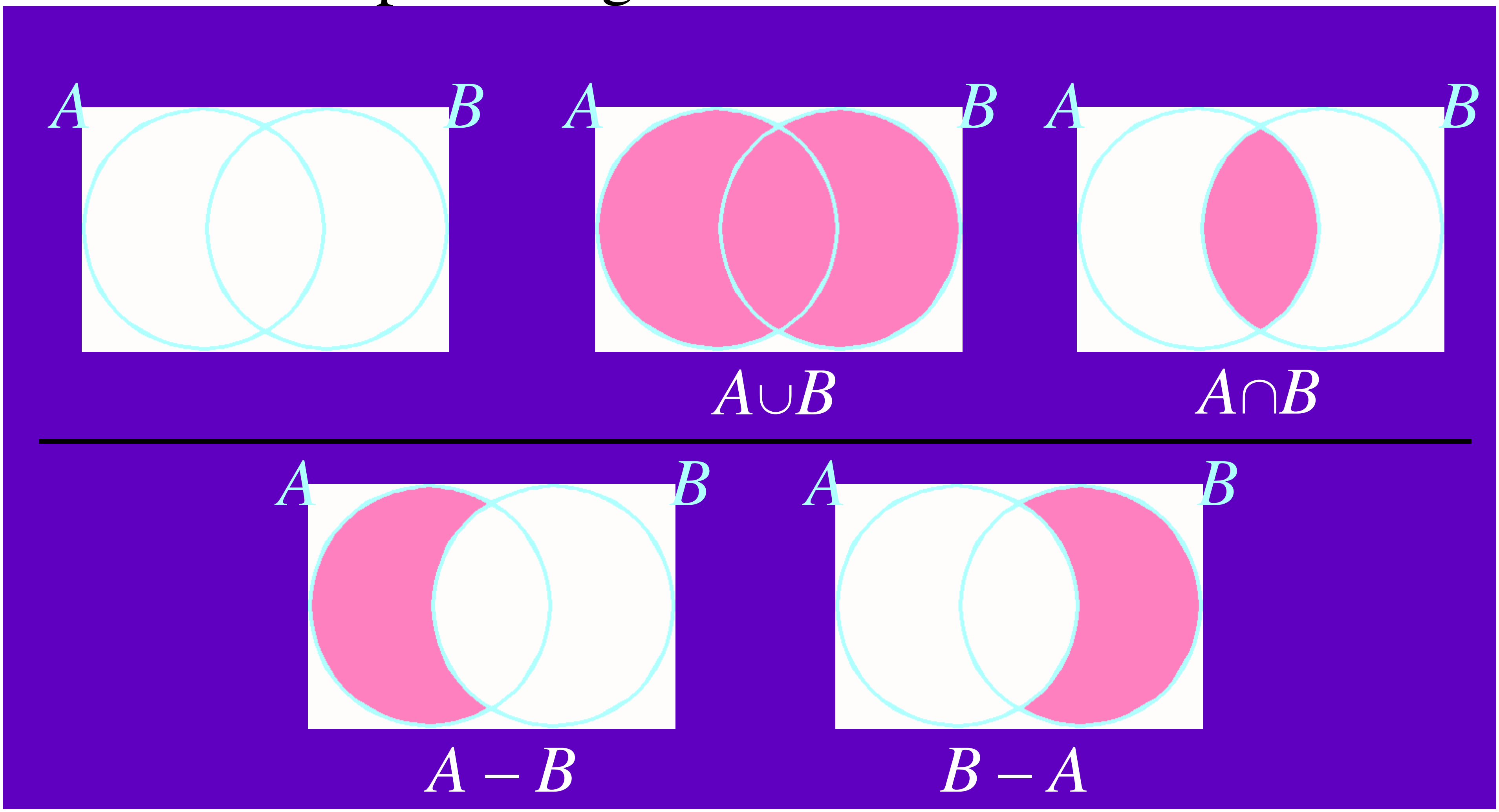
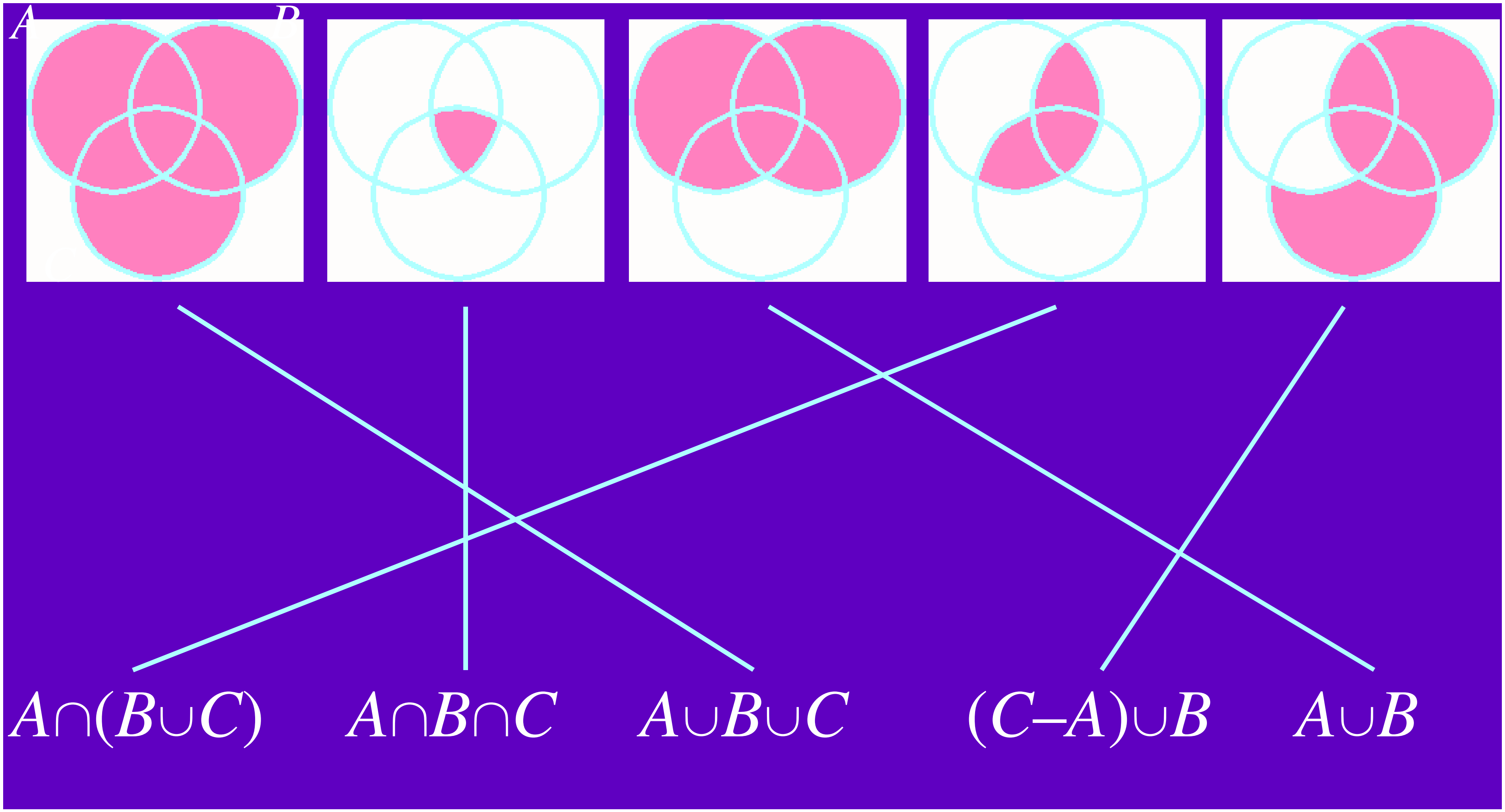
Venn Diagrams

- Venn diagrams are used to visualize sets and relationships among sets



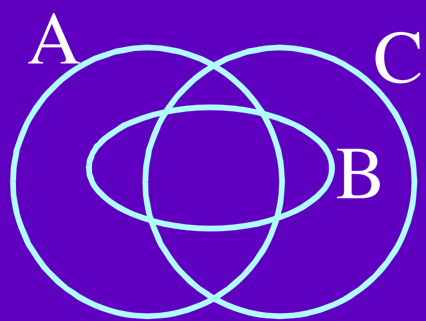
More Venn Diagrams



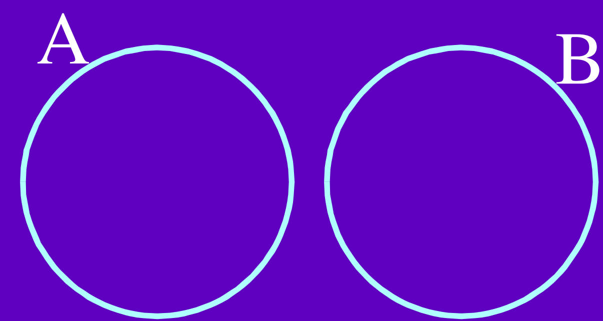
Venn Diagrams and Set Relationships

- Venn diagrams can be used to depict relationships between sets

C is a subset of *A*



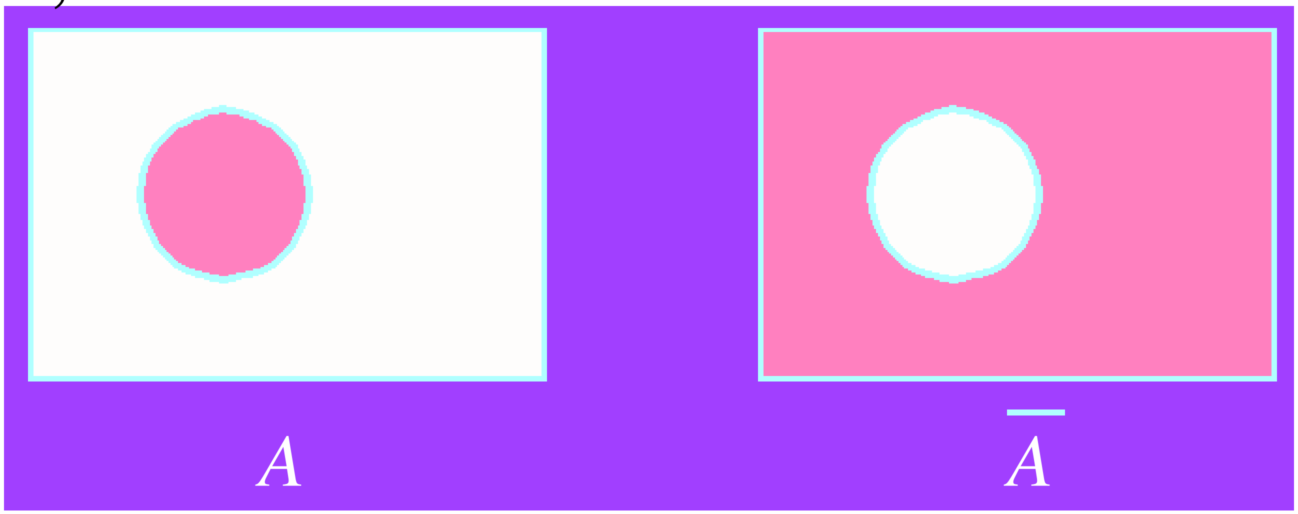
B is a subset of $A \cup C$



A and *B* are disjoint, that is, $A \cap B = \varnothing$

Venn Diagram of a Complement

- Recall that before defining the complement of a set we need a notion of *what*?
  - The universal set
- In a Venn diagram, we typically put a box around the whole figure to denote the universal set
- Then the complement of a set is the region inside the universal set, but outside the set we are considering



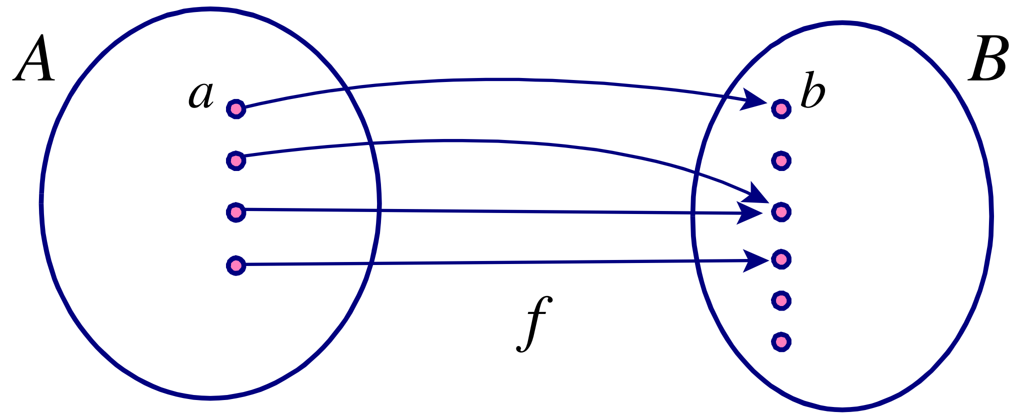
New Topic: Functions

But First...

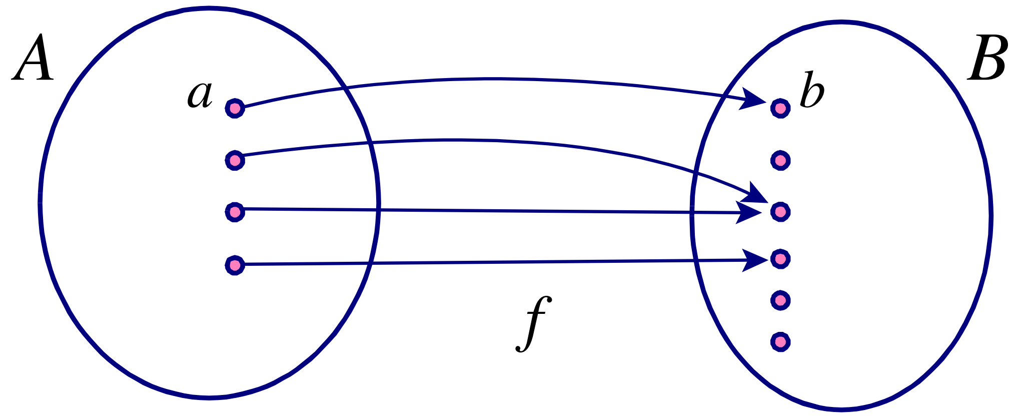
Any Questions?

Definition of Function

- Let A and B be any two sets.
- A function f:$A \to B$ is an assignment to each element of A exactly one element of B.
- If a is an element of A and f assigns b in B to a, then we write $f(a) = b$.
- We say that "f maps a to b," and that "b is the image of a under f."

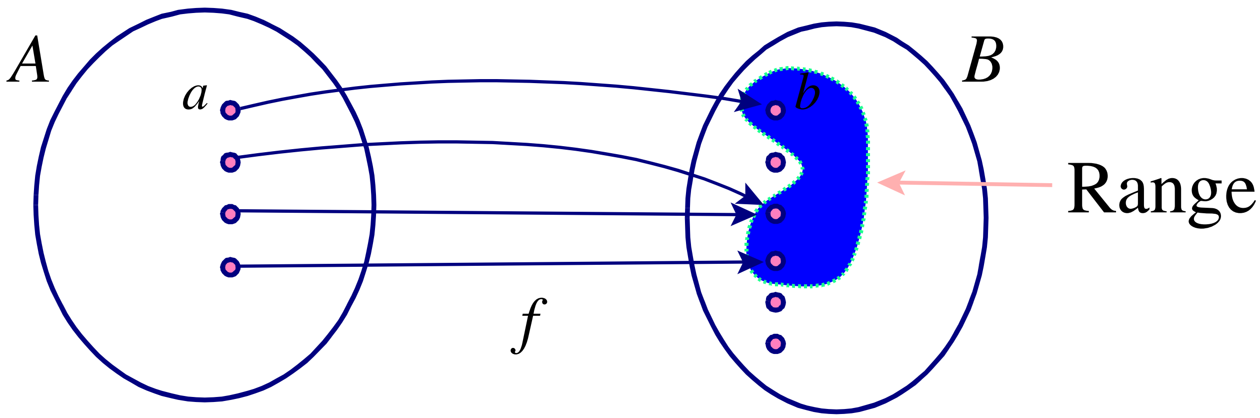


Some Notes on the Figure



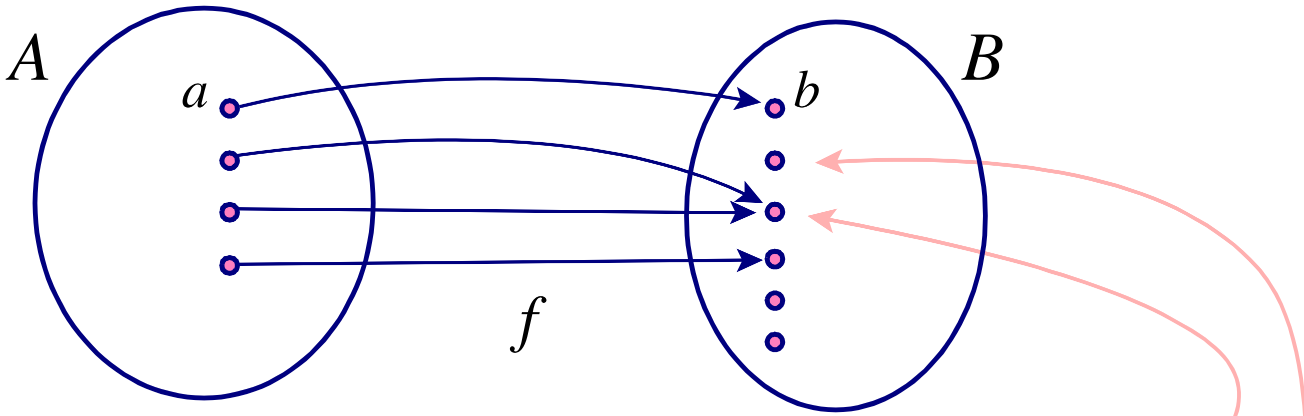
- *Every* element of *A* is assigned a value from *B*
- No value of *A* is assigned more than one value
- Not all elements of *B* are the image of some element of *A*
- It is okay that more than one element of *A* maps to the same element of *B*

Domain, Codomain and Range



- A is called the *domain* of f
- B is called the *codomain* of f
- The *range* of f is the set of elements of B that are the image of some element of A
  - range of $f = \{b \in B \mid f(a) = b \text{ for some } a \in A\}$
  - range of $f = \{f(a) \mid a \in A\}$

One-to-one and Onto



- f is called *one-to-one* if that doesn't happen:
  - f is 1-1 if: $f(x) = f(y) \rightarrow x = y$
  - f is 1-1 if: $x \neq y \rightarrow f(x) \neq f(y)$
- f is called *onto* if that doesn't happen:
  - f is onto if: $\forall b \in B\ \exists a \in A\ (f(a) = b)$

Some Familiar Functions

- f: $\mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2 + 2$
  - What is the domain?
    - $\mathbb{R}$
  - What is the codomain?
    - $\mathbb{R}$
  - What is the range?
    - All real numbers greater than or equal to 2
  - Is f 1-1?
    - Nope: $f(1) = f(-1) = 3$
  - Is f onto?
    - Nope: We can never find an x so that $f(x) = 0$, for example

Some Familiar Functions

- f: $\mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 - x$
  - What is the domain?
    - $\mathbb{R}$
  - What is the codomain?
    - $\mathbb{R}$
  - What is the range?
    - $\mathbb{R}$
  - Is f 1-1?
    - Nope: $f(-1) = f(0) = f(1)$
    - fails the *horizontal line test*
  - Is f onto?
    - Yes