Negating Expressions

- We use DeMorgan's laws to negate conjunctions and disjunctions:
  - $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
  - $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$
- And to negate quantifiers, we use the following:
  - $\neg\forall x\ P(x) \Leftrightarrow \exists x\ \neg P(x)$
  - $\neg\exists x\ P(x) \Leftrightarrow \forall x\ \neg P(x)$
- For example, we simplify: $\neg(\exists x\ P(x) \vee \forall x\ Q(x)\ )$
  - $\neg(\exists x\ P(x) \vee \forall x\ Q(x)\ ) \quad \Leftrightarrow \neg\exists x\ P(x) \wedge \neg\forall x\ Q(x)$
  - $\Leftrightarrow \neg\exists x\ P(x) \wedge \neg\forall x\ Q(x) \ \Leftrightarrow \forall x\ \neg P(x) \wedge \exists x\ \neg Q(x)$
- Compare first and last expressions if P(x) means "x is a dictator" and Q(x) means "x is free"

Sets

- Sets are collections of objects.
- The objects in a set are called its *elements*. If a is an element of set A, we write $a \in A$
- A set can be described in two ways:
  - Explicitly, by listing its elements within curly braces:
    - {a, b, c, d, e}
    - {Raleigh, Albany, Austin, ..., Sacramento}
    - {1, 2, 3, Albert Einstein, North Pole, 4, 5}
  - With *set builder* notation
    - $\{x \mid x > 10\}$
    - $\{p \mid p$ is prime and p is one more than a multiple of $4\}$
    - $\{a \mid a$ is a subset of $\{1, 2, 3\}\ \}$
  - Note the need for a universe of discourse when using set builder notation

The Empty Set

- There is a unique set which contains no elements
- It's called the *empty set*
- It is denoted by "$\varnothing$"
- ...or "{ }"

- It is a deceptively important set. Don't forget about it!
- It's the set analogue of the number "0"

Cardinality of a Set

- The *cardinality* of a set is simply the number of elements of that set
- For a finite set with n elements, the cardinality of that set is n
- If a set does not have a finite number of elements, then we say it is of infinite cardinality
  - We will see later that some infinities are bigger than others
- We denote the cardinality of a set S by $|S|$
- What is $|\varnothing|$?
  - 0

Equality of Sets

- Two sets are equal if they have exactly the same elements

- Note that this implies that if two sets are equal, then the have the same cardinality

- Thus, the following pairs are not equal:
  - $\{1, \{2\}\} \neq \{\{1\}, 2\}$
  - $2 \neq \{2\}$
  - $\{x \mid x^2 = 4\} \neq \{2\}$

Subsets

- If A and B are two sets, then we call A a *subset* of B if every element of A is also an element of B
  - $A \subseteq B \leftrightarrow (x \in A \rightarrow x \in B)$
- We write $A \subseteq B$
- If $A \subseteq B$, but $A \neq B$, then we can write $A \subset B$ to denote that A is a *proper subset* of B

Let $S = \{1, 2, 3\}$ $\quad T = \{1, 2, 3, 4\}$
$U = \{0, 1, 2, 3\}$ $\quad V = \{y \mid y \text{ is an integer} < 4\}$

Which of the sets above are subsets of which other sets? Can you suggest a nice way to picture these subset relationships?

The Power Set

The *power set* of a set *A is* the set of all subsets of *A*

- The power set is a set all of whose elements are also sets
- Find the power set of each of the following:
  - {1}
  - {a, b}
  - {α, β, γ}
  - $\varnothing$