Probability Problems

1. What is the probability that a fair coin tossed 4 times comes up heads each time?

2. A fair coin is tossed 10 times, and comes up heads each time. What is the probability that on the 11^{th} toss, it comes up heads again?

3. Five coins are tossed in succession and the result is recorded, as in "HHTHT." What is the probability that you never get an "H" immediately after a "T"?

4. A girl has one gummy bear in her hands, and a bowl of them on the table. She tosses a coin 6 times. If she gets Heads, she takes a gummy bear from the bowl, and if she tosses Tails, she puts one back into the bowl. The deal is that if she ever has more than 1 gummy bear, the game immediately ends and she gets none at all; and if she ever tosses Tails when she has no gummy bears, the game immediately ends and she gets none at all. What is the probability that she makes it to the end of the game and gets to keep her gummy bear?

5. **(Extra Credit, tricky problem)** A certain carnival game consists of a spinner with 30 sectors with various payoffs. Two of the sectors win $10, five of them with $5, ten of them win $2, nine of them win $1 and the rest are losers. How much should the carnival charge to play this game?

6. A die is rolled to yield a number from 1 to 6, and then a coin is tossed that many times. What is the probability that heads never appears?

7. (*Before solving this problem, take a guess as to which, a. or b., will yield a greater probability.*)
   Bill's three sons were all born in the same month (in different years). What is the probability of this:
   a. assuming that each month is equally likely?
   b. assuming that each day of the year is equally likely (ignoring Feb. 29^{th} for simplicity)?

8. **(Extra Credit, tricky problem)** A coin is tossed n times. Show that, regardless of the value of n, the probability of getting an even number of heads is exactly 1/2.

9. Ten balls are in an urn; five red and five blue. When a red ball is removed, it is always replaced. When a blue ball is removed a coin is flipped. If heads appears the ball is not replaced. If tails appears, then two blue ones are placed into the urn. What is the probability that the first three balls drawn are the same color?

Complement Probability

It is often easier to find the probability of something *not* happening than to find the probability of it happening. This is useful, because if p is the probability of an event happening, and q is the probability of it not happening, then $p + q = 1$.

For example, suppose we wish to find the probability that when a die is rolled three times a six will appear at least once. Finding this probability directly is tedious. So let's consider the complement probability, that is, the probability that a six will never appear. On each toss, the probability that a six does not appear is 5/6, so the probability that a six never appears is $(5/6)^3$. Thus the probability that a six *will* appear is $1 - (5/6)^3$.

10. A man rolls a pair of dice 24 times, and wins if he ever gets boxcars (double sixes; sum = 12). What is the probability that he wins?

11. For this problem, assume that there are 365 possible birthdays, and that each is equally likely.
    a. What is the probability that there are two people in this room with the same birthday?
    b. What is the largest number of people that can be in a room so that the probability of two having the same birthday is less than 50%?
    c. What is the smallest number of people that can be in a room so that the probability of two having the same birthday is greater than 50%?

12. It's been said that if you put a billion monkeys in front of a billion typewriters for a billion years, they will eventually type the complete works of Shakespeare. Let's consider this... Suppose that our billion monkeys have keyboards with just capital letters, three punctuation symbols and a space bar, and that each is typed with equal probability.
    a. Their first task is to type

       TO BE OR

       alone on a single sheet of paper. Assume they have been trained to insert a page, type 8 (random) characters, and remove the page. What is the probability that one of our billion monkeys will get this right on the first try?
    b. What is the probability that one of the monkeys will get it right within the first 1000 tries?
    c. Now suppose that each monkey can type twelve characters per second, and that putting paper into and taking paper out of a typewriter takes no time. How long would it take before the probability that one of our monkeys typed

       TO BE OR NOT

       alone on a single sheet of paper exceeded 50%? Assume they have been trained to insert a page, type 12 (random) characters, and remove the page. For this problem, you may wish to use the approximation $(1 + x)^n \approx e^{nx} \approx 1 + nx$, valid for small x, positive or negative.
    d. How about

       TO BE, OR NOT TO BE. THAT IS THE QUESTION.

       after they are retrained to type 43 characters on the page?