



Looking at the cycle structure:

(15847263)

$c = 1$

$n = 8$

$n - c = 7$  ...thus 7 equals the minimum number of rungs needed to create this permutation.

As one can see all of these methods give odd numbers, which it shall always do with any odd permutation. This is because each of these methods determines parity.

So there is not one but many ways to determine parity. The different ways to find parity are listed below:

### **The parity of a permutation is the parity of....**

- 1) The number of inversions
- 2) The number of swaps needed to get the permutation back to home position.
- 3) The number of rungs in any permutation ladder that generates  $\pi$ 
  - 3a)  $n - c$  \***equation for the minimum number of rungs\***  
... $n$  = the number of elements in the permutation and  $c$  = the number of cycles.

### **8-puzzle**

Imagine an 8 by 8 block puzzle with numbers in each block except for the bottom right:

6	4	2
7	3	5
8	1	

The first question that may come to mind is “Is this solvable?”  
In order to answer this question, our notion of parity is needed.

### **Theorem:**

An 8-puzzle (or any  $(ab - 1)$  puzzle for that matter....  $a$  and  $b$  being the width and height of the puzzle) is solvable if and only if the given permutation is even.

### **Proof of this theorem:**

Imagine the puzzle on a checker board and that the blank positioned in the bottom right-hand corner of the puzzle is the letter “L”.

6	4	2
7	3	5
8	1	L

A legal move can be thought of as a swap with “L”. “L” starts at the bottom-right black colored square and ends at the same black bottom-right square. Each time you perform a swap, the square on which “L” lies changes colors. This means “L” is on a black square every even move, and a white square every odd move. Since “L” ends on a black square, it will take an even number of moves to solve the puzzle. Which means it will take an even number of swaps to solve the puzzle. Thus only even permutations are solvable.