Discrete Math---Class 21---March 6, 2004

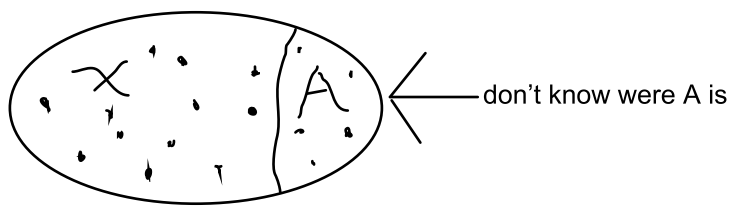
At the being of class we took at quiz and then discussed about some different types of sets.

Empty Set

Definition: Ø={} the set with no elements.

Therom: ∀A, Ø⊆A   A is a set

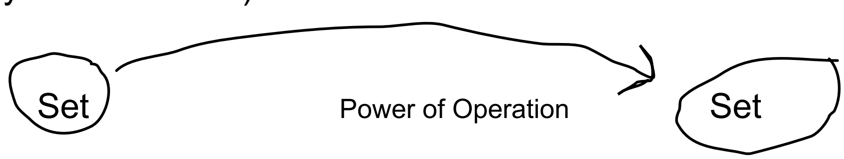
Proof: ∀x(x∈Ø→x∈A) True Statement
x∈U



- A propostion of the form ∀x (P(x) →Q(x)) is called vacuously satisfied or vacuously true if P(x) is false for all values of x.

Power Set

Definition: if A is a set, then the power set of A, denoted 2^A, or P(A), is the set of all subsets of A. (note: you can take the power set of only a set and it will give you another set)



Example: A={1,5"}
P(A)={{1},{5},{1,5}} the sets are subsets of A

Example: find P({a,b,c})
P({a,b,c})={ Ø,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}}

Example: taking the power set of a power set P(P({7}))
P(P({7}))=P({Ø,{7}})
={ Ø,{ Ø},{{7}},{ Ø,{7}}}

- Note that {{7}} is not the same as {7}
  - This set has 1 element namely {7}.
  - This set has 1 element namely 7.

- Sometimes, it is helpful to list a sets elements explicitly, when answering an apparently tricky question.
  - Example: is $\{2,\{3\}\} \subseteq \{ \emptyset,\{2,3\},\{2\}\}$
    - Elements of {2,{3}} are 2 and {3}
    - Elements of { Ø,{2,3},{2}} are Ø,{2,3}, and {2}
    - It is not a subset because the elements on the left side are not on the right side of the equation.

<u>Important Fact about Sets</u>: the elements in a set have no intrinsic order.
Thus {1,5}={5,1}
{5,4,3,2,1}={1,5,2,4,3}

<u>Cardinality</u>

<u>Definition</u>: let A be a set, and |A | means " the size of A" , and is the number of elements of A.

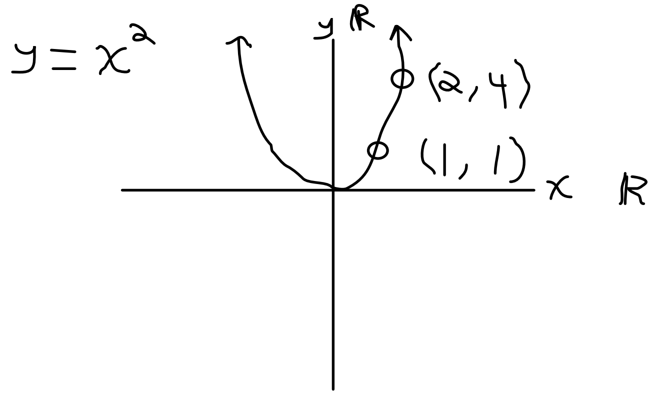
Example: |{2,3,4} |=3 Example: | Ø|=0 Example: |{2,3,4,{4,3,2}|=4

Example: "|A|≤100" says "A has at most 100 elements"

- Fact: $|P(A)| = 2^{|A|}$

<u>Cartesian Product of Sets</u>

<u>Definition</u>: If $A_1, A_2, \ldots, A_K$ are sets, then $A_1 \times A_2 \times \ldots \times A_K$ is the set of ordered K-tuples $(a_1, a_2, a_3, \ldots a_k)$, where $a_1 \in A_1, a_2 \in A_2, \ldots a_k \in A_k$.



IR X *IR* = IR^2 = {{x,y}|xЄ*IR*, yЄ*IR*}

Example: A={2,3}
B={John,Sally,Bill}
A x B={(2,John),(2,Sally),(2,Bill),(3,John),(3,Sally),(3,Billy)}

- Matter how you write the pairs.
- Example: (2,John) is not the same as (John,2)