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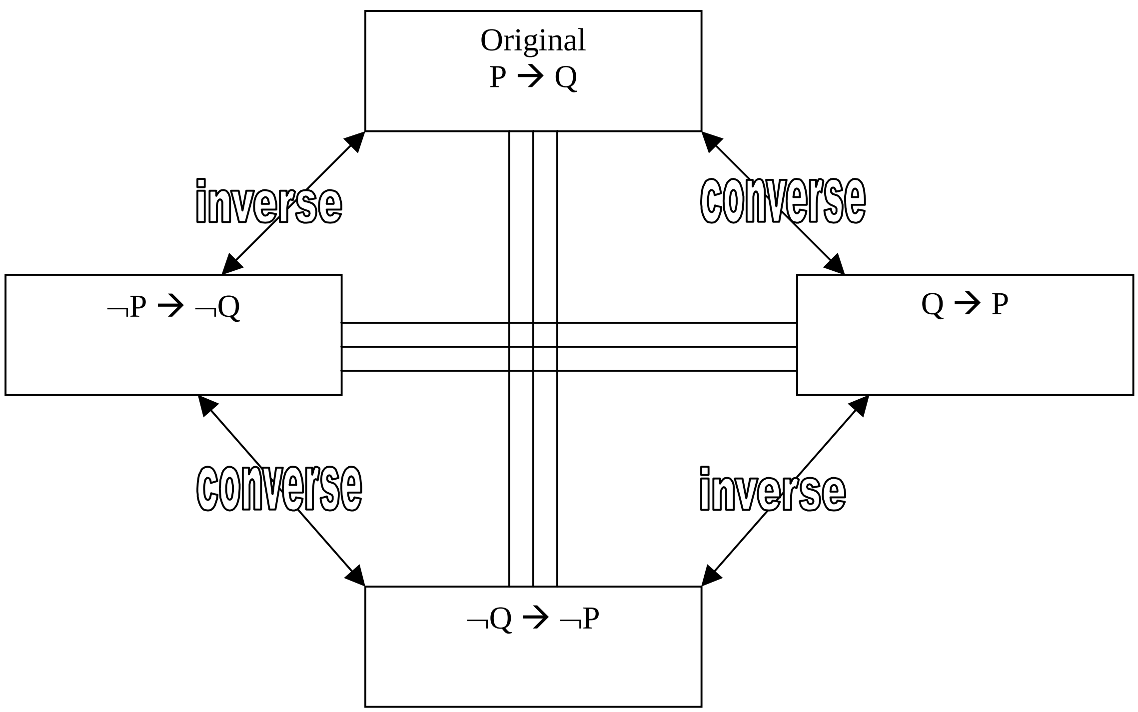
Notes for 3/2

- Started class by answering questions about the homework due on 3/4/04
  - A question about contrapositives:
    - A contrapositive is a variation on an implication
    - Original: $P \rightarrow Q$ = if you study, then you will get an A
      - P = you study
      - Q = you will get an A
    - 3 types of variations
      - inverse: is a negation of the original; $\neg P \rightarrow \neg Q$
        - if you do not study, then you will not get an A
      - converse: a switch of the original; $Q \rightarrow P$
        - if you got an A, then you studied
      - contrapositive: is a switch and negation of original;
        - $\neg Q \rightarrow \neg P$
        - if you did not get an A, then you did not study



    - The original and contrapositive will **always** be logically equivalent to each other
    - The inverse and converse of the original will **always** be logically equivalent to each other

        - Example: Original = x^2 is odd if x is odd
            - Contrapositive = x is not odd if x^2 is not odd; or if x^2 is not odd, then x is not odd.
    - Implications that have a "whenever" in them:
        - Example: I don't go to class whenever it rains. ($Q \rightarrow P$)
            - P = I don't go to class
            - Q = it rains
            - Converse: $P \rightarrow Q$, if I don't go to class, then it rains
            - Contrapositive: $\neg P \rightarrow \neg Q$, if I do go to class, then it doesn't rain.

    - Hypothetical Syllogism and Disjunctive Syllogism
        - H.S. = $[(P \rightarrow Q) \Lambda (Q \rightarrow R)] \rightarrow (P \rightarrow R)$
        - D.S. = $[(P \vee Q) \vee \neg P] \rightarrow Q$

- Nested Quantifiers
    - $\forall$ = all
    - $\exists$ = some
    - Read from left to right
        - Examples: P(x,y): x + 3 = y; x and y are natural numbers(universe of discourse)
            - $\forall x \forall y P(x, y)$ should be read $\forall x[\forall y P(x, y)]$; for every x, and for every y, x + 3 = y. Answer will be false
            - $\forall x[\exists y P(x, y)]$ – for all x, is the stuff in brackets true?
                - X = 5: $\exists y P(5, y)$; $\exists y$ 5 + 3 = y
                - X = 1: $\exists y P(1, y)$; $\exists y$ 1 + 3 = y
            - $\exists x[\forall y P(x, y)]$ = Does there exist an x so that for all y, x + 3 = y, will be false.
        - How to prove that:
            - $\exists x P(x)$ is true: find one example
            - $\exists x P(x)$ is false: requires some argument
            - $\forall x P(x)$ is true: requires some argument
            - $\forall x P(x)$ is false: fine one example
        - More examples:
            - P(x): x2 – 1 is a multiple of 3, (x is a natural number)
                - $\forall x P(x)$ = false, consider x = 0
                - $\exists x P(x)$ = true, consider x = 5
            - P(x): x2 + x is even, (x is a natural number)
                - $\forall x P(x)$ = true, but needs argument
                - $\exists x P(x)$ = true, consider x = 3
            - Q(n): n2 + 7n + 10 is a prime number, (x is a natural number)
                - $\forall x Q(n)$ = false, but needs an argument
                - $\exists x Q(n)$ = false, consider n = 1

- Try this at home:
- R(x): x2 + x + 41 is a prime, (x is a natural number)
    - $\forall x\, R(x) =$
    - $\exists x\, R(x) =$

- We started to go over negation, but only did two:
    - $\neg(\forall x\, P(x)) = \exists x\, \neg P(x)$
    - $\neg(\exists x\, P(x)) = \forall x\, \neg P(x)$
    - will go over the rest on 3/4/04