

Discrete Mathematics – Class 1 – Tuesday, January 13, 2004

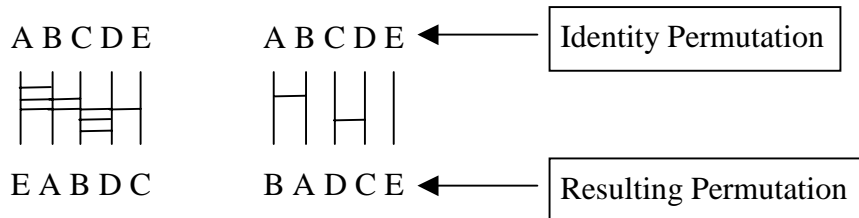
A permutation is a (re) arrangement of some set of objects.

We will typically be given some set of objects in some initial arrangement (called the *home position*, or *identity permutation*) and will be rearranging them into some other order.

The starting arrangement is called the “identity permutation”

Permutation Ladders can be used to generate permutations. To generate a permutation, we “trace” each object from its position at the top (its “home position”) down the wires to the bottom of the diagram. Each time we encounter an endpoint of a *rung* of the ladder, we must trace to the other end of that rung before continuing down the diagram.

Here are two examples of permutation ladders on the set of objects {A, B, C, D, E}:



It is possible to have a permutation ladder with no rungs. Such a permutation ladder is said to generate the “identity permutation,” because every object will simply drop straight down the diagram to the bottom.

We spent some time considering the following questions:

1. Is it possible for the last person who draws a rung to draw that rung at the bottom in such a way to force himself to be the last note-taker?
 - a. Yes: He simply finds where he ended up, and then adds a rung from that position to the last position.
2. Can he add a rung at the top of the diagram to place himself last in the note-taking order?
 - a. Yes: Trace from the last position at the bottom of the diagram back *up* to the top of the diagram to find out who ended up last. Then add a rung connecting that position to his own name. In this fashion, he “hijacks” the route to last place.
3. Suppose the teacher specifies some “altitude” at which the line must be added. Is it possible for a player to add a rung at that altitude so that he again ends up in last position?
 - a. Again, yes. This is possible. To do so, trace the last place position *up* to that altitude to find one vertical line, and then trace that player’s name from the top of the diagram *down* to that altitude to find a second vertical line. Then add a rung at the desired altitude, and the chosen player will in this fashion “hijack” the route to the bottom of the diagram.

Some homework questions:

1. Is there a permutation ladder which generates the permutation "54321" from the home position "12345"?
2. What is the least number of rungs needed in such a ladder?
3. What is the least number of rungs needed to generate the permutation "321" from the home position "123"?
4. In general, what is the least number of rungs needed to reverse a permutation on n elements?
5. Prove that given any permutation on any number of objects, there is some permutation ladder which generates that permutation.

Note-taking order

1. Troy
2. Tyler
3. Chris
4. Sam
5. David
6. Danny
7. Brian
8. Teedra
9. Kenny
10. Duran
11. Beatrice
12. Viktor
13. Adam
14. Josh
15. Jerry
16. Glendolyn
17. Clarissa
18. Ernest
19. Randy