Last Day:

Anagram Formula (on final)

\# anagrams of a word is $\dfrac{n!}{k_1! \cdot k_2! \cdot \ldots \cdot k_r!}$

where n = total # letters

k_i = # of times each letter is repeated.

\# anagrams of LOVE $= \dfrac{4!}{1! \cdot 1! \cdot 1! \cdot 1!} = 4!$

MISSISSIPPI $= \dfrac{11!}{1!\,4!\,4!\,2!} = \dfrac{11 \cdot 10 \cdot 9 \cdot \cancel{8} \cdot 7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{1} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1} \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1} \cdot \cancel{2} \cdot \cancel{1}}$

M I S P

$= 11 \cdot 10 \cdot 9 \cdot 7 \cdot 5 = \boxed{34{,}650}$

$$\text{MOM} = \frac{3!}{2!} = \frac{3\cdot\cancel{2\cdot 1}}{\cancel{2\cdot 1}} = \boxed{3}$$

MMO
MOM
OMM

$$\text{TEETER} = \frac{6!}{2!\,3!} = \frac{6\cdot 5\cdot \cancel{4}^{2}\cdot\cancel{3\cdot 2\cdot 1}}{\cancel{2\cdot 1}\cdot\cancel{3\cdot 2\cdot 1}} = \boxed{60}$$

Formula for $\boxed{\binom{n}{k} = \frac{n!}{k!\,(n-k)!}}$ the end.

$$\underbrace{YYY\cdots Y}_{k}\underbrace{NNN\cdots N}_{n-k}$$

Let's compute $\binom{8}{3}$ = # ways to select 3 items out of 8 distinct items.

# of sets of size 3 = # of ways to answer these 8 questions

A	B	C	D	E	F	G	H
Y	N	N	N	Y	N	N	Y
N	N	Y	N	Y	Y	N	N

$\Longleftrightarrow \{A, E, H\}$ = # of anagrams of YYYNNNNN $= 8!/3!\cdot 5!$

$$\binom{8}{3} = \frac{8!}{3!\,5!} = \frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}}{\cancel{3}\cdot \cancel{2}\cdot 1\cdot \cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}} = \boxed{56}$$

$$\binom{108}{2} = \frac{108!}{2!\cdot 106!} = \frac{\overset{54}{\cancel{108}}\cdot 107\cdot \cancel{106!}}{\cancel{2}\cdot 1\cdot \cancel{106!}}$$

$$\binom{n}{1} = \frac{n!}{1!\,(n-1)!} = \frac{n\,\cancel{(n-1)(n-2)\cdots}}{\cancel{(n-1)(n-2)\cdots}}$$

$$\begin{array}{r} 107 \\ \times\; 54 \\ \hline 428 \\ 535 \\ \hline \boxed{5778} \end{array}$$

Sample test questions

$$\binom{40}{2} \qquad \binom{20}{3} \qquad \binom{100}{2}$$

$$\binom{8}{4} \qquad \binom{9}{4} \qquad \binom{12}{3} \qquad \binom{n}{1} = n$$

$$\binom{n}{2} = \frac{n!}{2!\,(n-2)!} = \boxed{\frac{n(n-1)}{2}}$$

Final Study
- previous exams
- Assigned HW
- Answers to HW (5^ch will be set around)

Style will be the same as the exams.

No calculator No friends No cheating No bribes ..	No notes / technology.

Thm: n is divisible by 3 $\Longleftrightarrow$ sum of digits is divisible by 3

Proof: $n = \underbrace{d_k \cdots d_2 d_1 d_0}_{\text{digits}} = 10^k \cdot d_k + \cdots + 10^2 \cdot d_2 + 10 \cdot d_1 + d_0 \equiv d_k + \cdots + d_1 + d_0 \pmod 3$