Operating Systems II **Name:_____**
Exam II **November 21, 2006**

This exam has 9 questions. Please make sure that you have them all. Answer each question on a separate sheet of paper, and pay attention to the instructions, in italics, associated with each problem.

1. Show how to modify the Basic Algorithm from Korach's paper so that every node of a tree may discover, in a distributed fashion, the number of leaves in that tree. Analyze the total number of messages that must be sent before your algorithm terminates, and mention whether your algorithm is synchronous or asynchronous. *Do not use the Internet for this problem.*

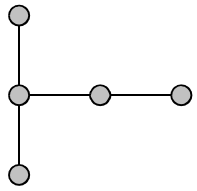
2. In class I mentioned that in a synchronous tree network, the Basic Algorithm of Korach could yield a single saturated vertex only. Prove this, by considering a proper two-coloring of the vertices of the tree. *You may use the Internet for any portion of this problem.*

3. Modify the Basic Algorithm of Korach to perform leader election on a general topology. *Do not use the Internet for this problem.*

4. In the paper of Fischer et al. on Distributed Consensus, the proof of Lemma 3 uses the possibility of a processor p becoming faulty at some point (see case 2 in the proof) and taking no further steps. But since the identity of processor p may change as the protocol is executed, isn't it possible that this proof is really requiring more than one processor to become faulty? Explain why this is not so.

5. Give an algorithm for ten folks in a conference call to come to agreement on the value of a computed bit, where up to four of the participants may have performed the computation in a faulty fashion.

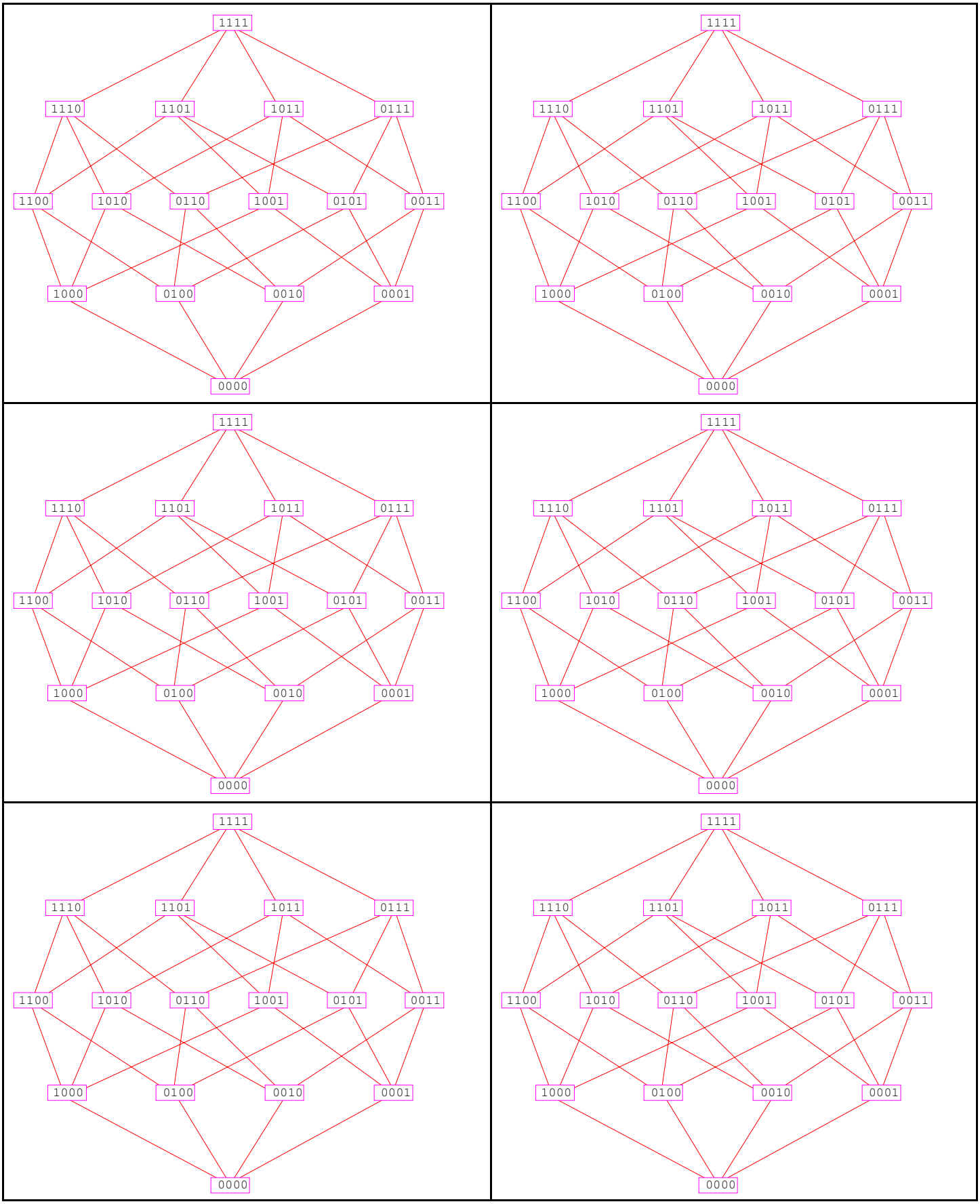
6. Find three different labelings of the nodes of the tree shown to the right, with binary strings of length 4, without stars, so that Hamming distance reflects network distance. Then find each of your labeled trees within a copy of the 4-cube shown on the following pages. (You need only three of the copies, so some extras are provided.) Be sure to indicate which copies of the 4-cube contain your solutions.



7. Show how the labeling of a K_4 network (4 nodes, all pairs connected) with the labels 1000, 11**, 10*1, 0***, arises from a squashing of the 4-cube. That is, select a copy of the 4-cube from the last two pages, and show which sub-cubes get squashed.

8. Find a labeling of the vertices of a 5-node ring topology with strings of length 4 so that network distance equals string distance. Show the corresponding picture on a 4-cube.

9. Find a labeling of the vertices of a 8-node ring topology with strings of length 4 so that network distance equals string distance. Show the corresponding picture on a 4-cube.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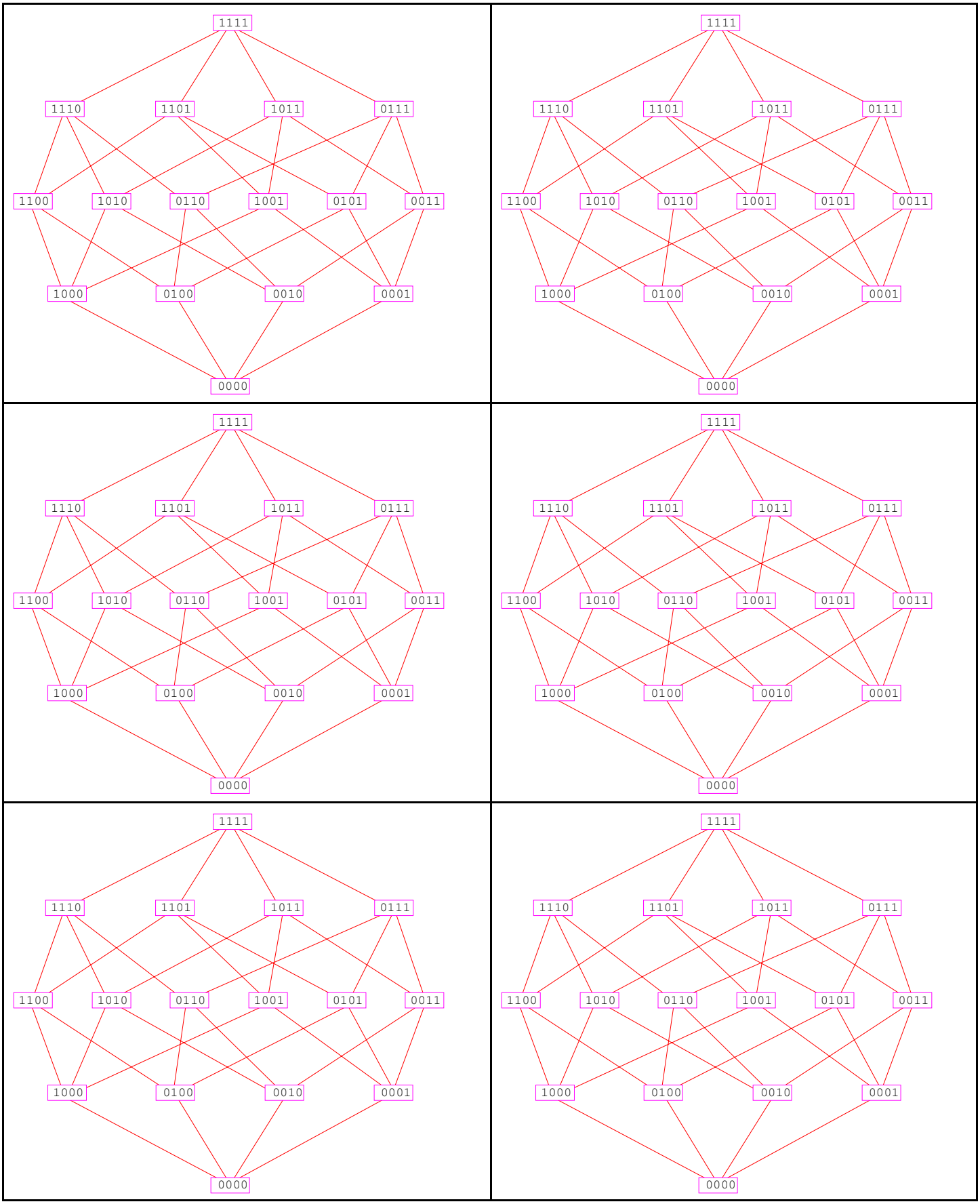
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