# Some Assumptions from the Play

- The secretary was neither gnawed by rats nor clawed by cats.
- If a person smelled like Limburger, then that person was clawed by cats.
- If a person was in the faculty lounge, then that person would smell like Limburger.
- A person either didn't change his socks or he was in the faculty lounge at some point.
- A person with dry socks either didn't get his or her feet wet or changed his or her socks.
- Either the secretary's feet got wet or she was wearing shoes the whole time.
- Either the secretary took off her shoes or she entered the principal's office for a while.

# Statements

**Definition:**
A *statement* is a sentence which is unagmbiguously true or false

For Example:

**Statements:**
- George Bush is the president of the USA
- Jacques Chirac is the president of Thailand
- Mrs. Kerry is richer than Mr. Kerry
- All line segments have two endpoints
- All angles are less than 45 degrees
- Michael Andrew Flit of Parsons Ln is 5'11"

**Non-Statements:**
- Mike is tall
- $x = 13$
- Angle A is 22 degrees
- Segment X is longer than segment Y
- $p$ is prime

# Predicates

A *predicate* is an assertion involving an unknown quantity, called a *propositional variable*.

**Examples:**

- $x = 13$
- Angle A is 22 degrees
- Segment X is longer than segment Y
- $p$ is prime

For every propositional variable mentioned in a problem, a *universe of discourse* must be given for that variable, indicating the set of values under consideration for that variable.

**Examples:**

- $x$ is a real number
- A is an angle of some triangle
- X and Y are segments in the plane
- $p$ is a positive integer

# Predicates can Become Propositions

We can turn predicates into propositions by assigning a value to its variable(s).

**Examples:**
"$p$ is prime"
becomes a proposition if we let $p = 20$.

"The man is a millionaire"
becomes a proposition if we let "the man" be "Mike Tyson."

# Quantification of Propositional Variables

We can also turn a predicate into a proposition by "quantifying" its variables with either "there exists" or "for all."

**Examples:**
"*p* is prime"
becomes a proposition if we say:
"For every value of *p*, *p* is prime," or
"There exists a value of *p* such that *p* is prime."

"The man is a millionaire"
becomes a proposition if we say:
"Every man is a millionaire," or
"Some man is a millionaire."

Notice how in each case the resulting assertion is unambiguously true or unambiguously false.

Also note that there is, in each case, an underlying universe of discourse for the variables.

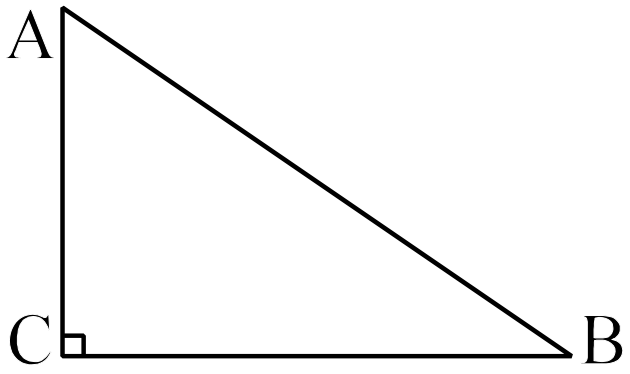
# Disjunctions

**Definition:**
A *disjunction* is the assertion that at least one of two given statements is true.

Notice that disjunctions are statements, because the truth values of the two given statements determines the truth value of the disjunction.

**For example:**

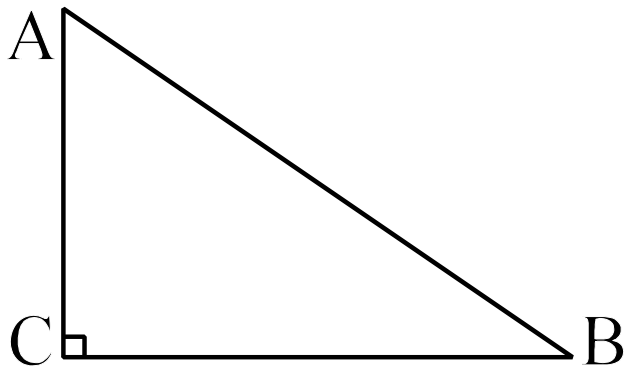
| | |
|---|---|
| Angle A is at least 45 degrees | True |
| Angle B is at least 45 degrees | False |
| Angle A is at least 45 degrees or Angle B is at least 45 degrees | True |



# Disjunctions

## Another Example:

| | |
|---|---|
| Angle A is 90 degrees | False |
| Angle B 120 degrees | False |
| Angle A is 90 degrees or Angle B is 120 degrees | False |




In this case the disjunction is false because each of its component statements is false.

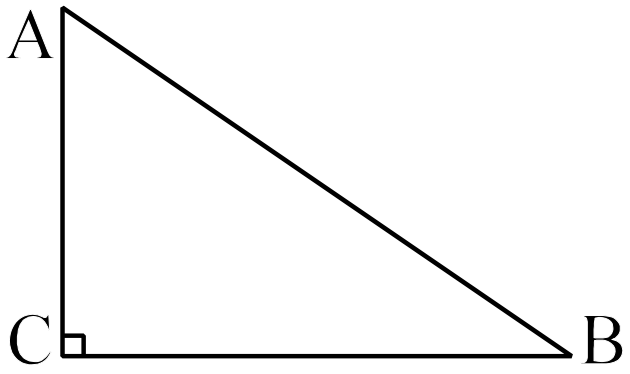
# Conjunctions

**Definition:**
A *conjunction* is the assertion that both of two given statements are true.

Notice that conjunctions are statements, because the truth values of the two given statements determines the truth value of the conjunction.

**For example:**

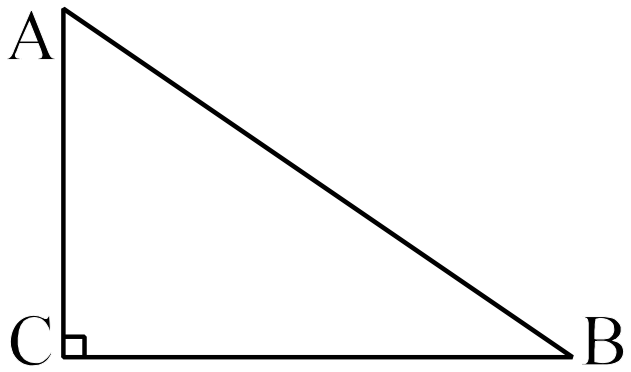
| | |
|---|---|
| Angle A is at least 45 degrees | True |
| Angle B is at least 45 degrees | False |
| Angle A is at least 45 degrees and Angle B is at least 45 degrees | False |



# Conjunctions

## Another Example:

| | |
|---|---|
| Angle A is less than 100 degrees | True |
| Angle C is 90 degrees | True |
| Angle A is less than 100 degrees and Angle C is 90 degrees | True |




In this case the conjunction is true because each of its component statements is true.

# Implications

**Definition:**
An *implication* involves two statements, called the *premise*, or *hypothesis*, and the *conclusion*. It is the assertion that if the premise is true, then the conclusion must be true. It makes no claim in the case that the premise is false.

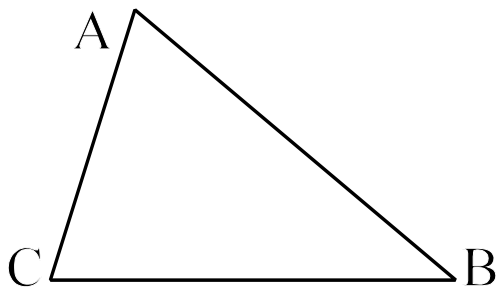
**Examples:**

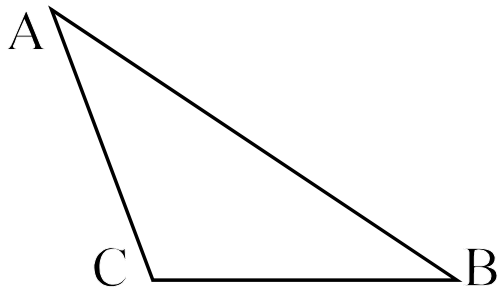
- If $1 + 2 = 3$, then $1 > 0$

This is true, because the premise is true and the conclusion is true

- If $\triangle ABC$ is acute, then $\angle CAB$ is less than 90°.

Here are two different triangles ABC:




The implication is true in both cases. In the second case, we say it is "vacuously" true.

# Implications

We rig implications to be propositions by calling them "true" whenever the premise is false. This turns out to be the natural thing to do.

# Better Way to Think of it

An implication is false if the premise is true but the conclusion is not.

**Example:**
What would make the following implication false:

- If you give me $100, then I will make sure you pass this class

It is demonstrably false only when the premise is true but the conclusion is false.

Otherwise we say the implication is true.

# Negations

The negation of a statement is the assertion that the statement is false.

Note that the negation of a statement is another statement, having the opposite truth value of the original statement.

**Example:**

Statement: John is tall.
Negation: John is not tall.

In English you can always form the negation of a statement by putting the phrase "It is not the case that" at the beginning. There are often more natural ways to negate a statement, though.

Statement: John is rich or John is handsome.
Negation: It is not the case that John is rich or John is handsome
Better: John is neither rich nor handsome

# Syllogisms

A syllogism is an abstract logical argument.

**Examples:**

1. If an implication is true and its premise is true, then its conclusion must be true.

2. If the disjunction of two statements is true and the first statement is false, then the second statement must be true.

3. If an implication is true and its conclusion is false, then its premise must be false.

4. If two implications are true, and the conclusion of the first is the premise of the second, then the premise of the first implies the conclusion of the second.

5. If the conjunction of two statements is true, then each of the statements is true.

# Syllogisms in Action

Here are two statements:

- The secretary was neither gnawed by rats nor clawed by cats.
- If a person smelled like Limburger, then that person was clawed by cats.

What can we conclude?

Observe that "nor" means "and also not," so that the first statement is really a conjunction of
"The secretary was not gnawed by rats," and
"The secretary was not clawed by cats."

Since the conjunction is true, we can conclude that each component statement is true. In particular, we can conclude that the secretary was not clawed by cats.

Then by syllogism "3" on the previous slide, we can conclude that the secretary did not smell like Limburger.

# Syllogisms in Action

Here are two statements:

- The secretary did not smell like Limburger.
- If a person was in the faculty lounge, then that person would smell like Limburger.

What can we conclude?

By the same reasoning, we can conclude that the secretary had not been in the faculty lounge.

Here are two statements:

- The secretary was not in the faculty lounge.
- A person either didn't change his socks or he was in the faculty lounge at some point.

By rule "2" on that slide, we can conclude that the secretary did not change her socks at any point.

# And So On...

- The secretary had dry socks and had not changed her socks.
- A person with dry socks either didn't get his or her feet wet or changed his or her socks.

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$\therefore$ The secretary did not get her feet wet

- The secretary did not get her feet wet
- Either the secretary's feet got wet or she was wearing shoes the whole time.

---

$\therefore$ The secretary was wearing shoes the whole time

- The secretary was wearing shoes the whole time
- Either the secretary took off her shoes or she entered the principal's office for a while.

---

$\therefore$ The secretary entered the principal's office.

- If the secretary was in the principal's office during the blackout, then she is the guilty party
- The secretary was in the principal's office

---

$\therefore$ The secretary is the guilty party

# The Symbols of Logic

We've already seen "$\therefore$" used to mean "therefore," signifying the conclusion of our argument.

It is also customary to express:

- Propositions with capital letters: P, Q, R, ...
- Propositional variables with lowercase letters in italics: $x$, $y$, $z$, ....
- Predicates as functions: P($x$), R($y$, $z$), ...
- Implications with arrows: $P \rightarrow Q$
- Disjunctions with this symbol: "$P \vee Q$"
- Conjunctions with this symbol: "$P \wedge Q$"
- Negation with this symbol: "$\neg P$"
- "For all" with "$\forall$" and "there exists" with "$\exists$"

# Symbolic Form of Syllogisms

| | |
|---|---|
| If an implication is true and its premise is true, then its conclusion must be true. | $P \rightarrow Q$<br>$P$<br>$\therefore Q$ |
| If the disjunction of two statements is true and the first statement is false, then the second statement must be true. | $P \vee Q$<br>$\neg P$<br>$\therefore Q$ |
| If an implication is true and its conclusion is false, then its premise must be false. | $P \rightarrow Q$<br>$\neg Q$<br>$\therefore \neg P$ |
| If two implications are true, and the conclusion of the first is the premise of the second, then the premise of the first implies the conclusion of the second. | $P \rightarrow Q$<br>$Q \rightarrow R$<br>$\therefore P \rightarrow R$ |
| If the conjunction of two statements is true, then each of the statements is true. | $P \wedge Q$<br>$\therefore P, Q$ |

# Names of Syllogisms

| | |
|---|---|
| Modus Ponens | $P \rightarrow Q$<br>$P$<br>$\therefore Q$ |
| Disjunctive Syllogism | $P \vee Q$<br>$\neg P$<br>$\therefore Q$ |
| Modus Tollens | $P \rightarrow Q$<br>$\neg Q$<br>$\therefore \neg P$ |
| Hypothetical Syllogism | $P \rightarrow Q$<br>$Q \rightarrow R$<br>$\therefore P \rightarrow R$ |
| Simplification | $P \wedge Q$<br>$\therefore P, Q$ |

Note: In some books, "Hypothetical Syllogism" is called "The Law of Syllogism," and what we are calling "syllogisms," they call "arguments."

# Other Syllogisms

| | |
|---|---|
| Resolution | $P \vee Q$<br>$\neg P \vee R$<br>$\therefore\ Q \vee R$ |