

Discrete Mathematics, Homework #4, part 1.

1. How many 10-digit numbers are there which don't use any digit more than once, where no odd digit ever follows an even digit, and where the "4" and the "5" are next to each other?
2. Pappy's Pizza offers seven different toppings for their pizzas. How many different ways are there to make a pizza, including "plain" and "the works?"
3. Consider the following variation on the "Standing Bosses" game:
Line up some players, sitting in chairs, facing the class, then iterate:
 - The boss is the second-to-rightmost sitting player
 - The boss's job is to stand up, and to tell all standing players to his right to sit downThe game is repeated for as long as possible
For example, with 3 players, the game goes like: 000 \rightarrow 010 \rightarrow 100 \rightarrow 110 and game over.
 - a. The game with 3 players had 4 positions altogether. How many positions does the game with 4 players have altogether?
 - b. How many positions do the games with 1, 2, and 5 players have?
 - c. How many positions does the game with n players have?
4. Consider the following variation on the "Standing Bosses" game:
Line up some players, sitting in chairs, facing the class, then iterate:
 - The boss is the rightmost sitting player who does not have a person standing to his *immediate* left
 - The boss's job is to stand up, and to tell all standing players to his right to sit downThe game is repeated for as long as possible
For example, with 3 players, the game goes like: 000 \rightarrow 001 \rightarrow 010 \rightarrow 100 \rightarrow 101 and game over.
 - a. The game with 3 players had 5 positions altogether. How many positions does the game with 4 players have altogether?
 - b. How many positions do the games with 1, 2, and 5 players have?
 - c. How many positions does the game with 15 players have?
5. Consider the following variation on the "Standing Bosses" game:
Line up some players, sitting in chairs, facing the class, then iterate:
 - The boss is the person to the left of the leftmost sitting player, unless the leftmost sitting player is the leftmost player, in which case the boss is the rightmost player.
 - The boss's job is to stand up, and to tell all standing players to sit downThe game is repeated for as long as possible. How many positions does the game with n players have?
6. Suppose we have some variant of the standing bosses game with the following properties:
 - a. There is some unique start position
 - b. A position consists of a row of players, fixed in number, each either sitting or standing
 - c. There is a rule which, at each step, determines the next position from the current position alone, without using any randomness or any outside input.Prove that such a game on n players must either continue indefinitely, or stop within 2^n steps.