

Expected Value

Expected Value is another word for “mean” or “average.”

Suppose we have a random variable R that takes on the values

$$\{x_1, x_2, x_3, x_4, \dots\}$$

with probabilities

$$\{p_1, p_2, p_3, p_4, \dots\}$$

respectively.

Then the expected value is given by the formula:

$$E[R] = p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4 + \dots$$

(Note: All lists could be finite or infinite.)

Ummm... Maybe we can try an example...

Expected Value Example

Toss a coin 4 times and let R denote the number of times heads appears. What is the expected value of R ?

Value of R	Probability	Product
0	1/16	0
1	4/16	4/16
2	6/16	12/16
3	4/16	12/16
4	1/16	4/16
	Total	32/16

Expected Value Example

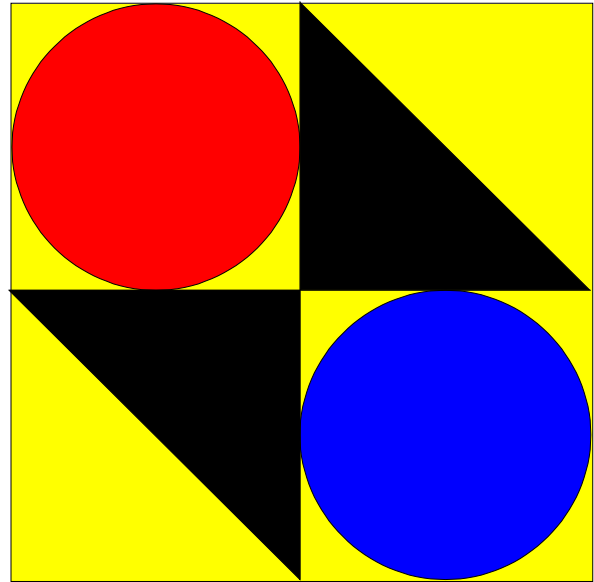
Bill draws a card randomly from a deck, and Bob gives him the number of dollars indicated by the rank (Ace counts “1,” face cards count “10”).

What is the expected payoff?

2♣	2♦	2♥	2♠
3♣	3♦	3♥	3♠
4♣	4♦	4♥	4♠
5♣	5♦	5♥	5♠
6♣	6♦	6♥	6♠
7♣	7♦	7♥	7♠
8♣	8♦	8♥	8♠
9♣	9♦	9♥	9♠
10♣	10♦	10♥	10♠
J♣	J♦	J♥	J♠
Q♣	Q♦	Q♥	Q♠
K♣	K♦	K♥	K♠
A♣	A♦	A♥	A♠

Expected Value Example

A carnival game consists of tossing a dart, which lands at a random spot within the square target to the right. The red circle wins \$5, the blue circle wins \$7 and either black triangle wins \$10.



If it costs \$5 to play this game, do you expect to make money or lose money?

Let R be the payoff:

Value of R	Probability	Product
5	$\pi/16$.98175
7	$\pi/16$	1.37445
10	1/4	2.50
	Total	4.8562

You expect to lose about 15 cents on each play.

Infinite Sum Expected Value Example

Toss a coin until heads appears, and let R be the number of tosses required. What is the expected value of R ? Try this at your desks...

Value of R	Probability	Product
1	1/2	1/2
2	1/4	2/4
3	1/8	3/8
4	1/16	4/16
5	1/32	5/32
...
n	$1/2^n$	$n/2^n$
	Total	

$$\text{Let } S = 1/2 + 2/4 + 3/8 + 4/16 + 5/32 + \dots$$

$$2S = 1 + 2/2 + 3/4 + 4/8 + 5/16 + \dots$$

$$2S - S = 1 + 1/2 + 1/4 + 1/8 + 1/16 + \dots$$

$$\text{So } S = 1/(1 - 1/2) = 2$$